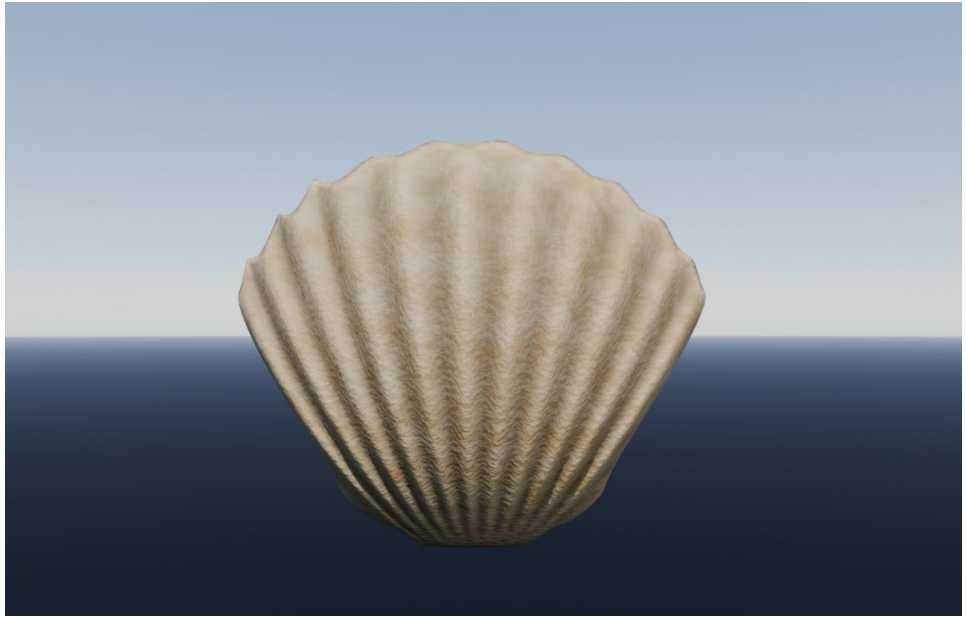
Simulation Part 1

Slides by Sarah Jobalia and Kevin Li, with modifications by Kate Baker

Student Reference Texture Highlights!



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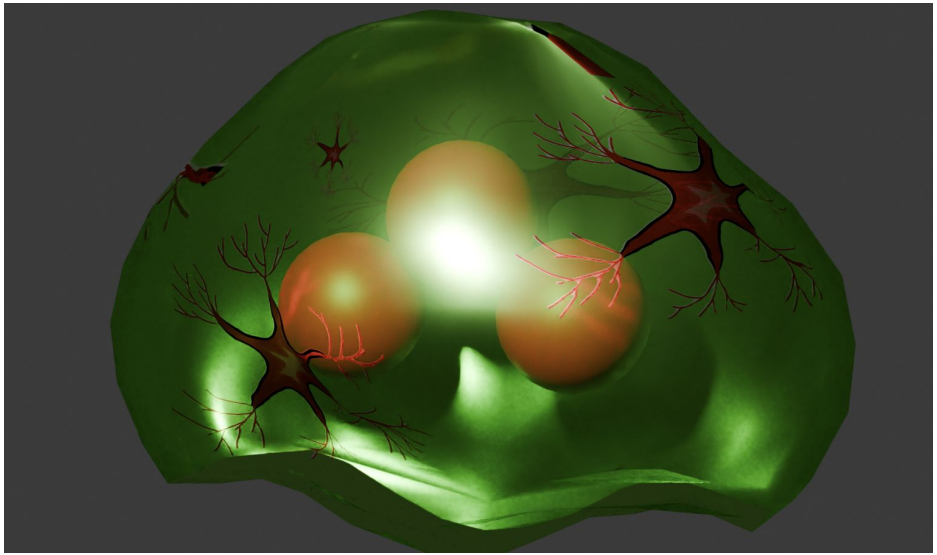
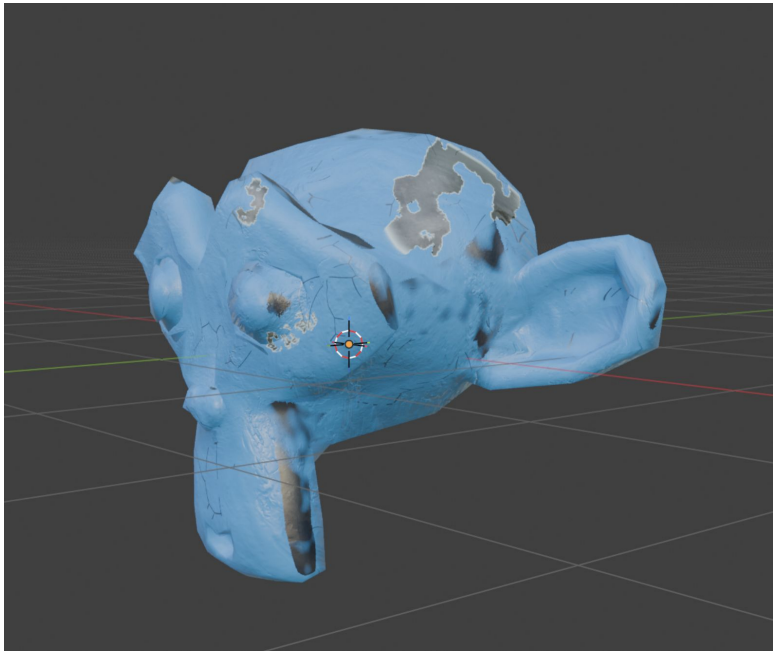


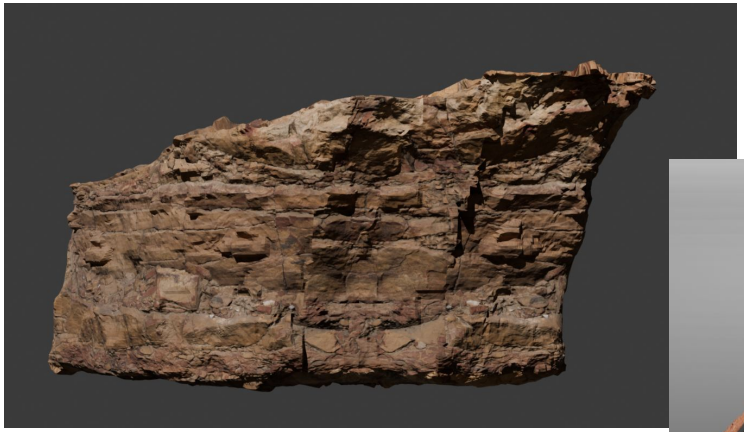
Zia Svendsen



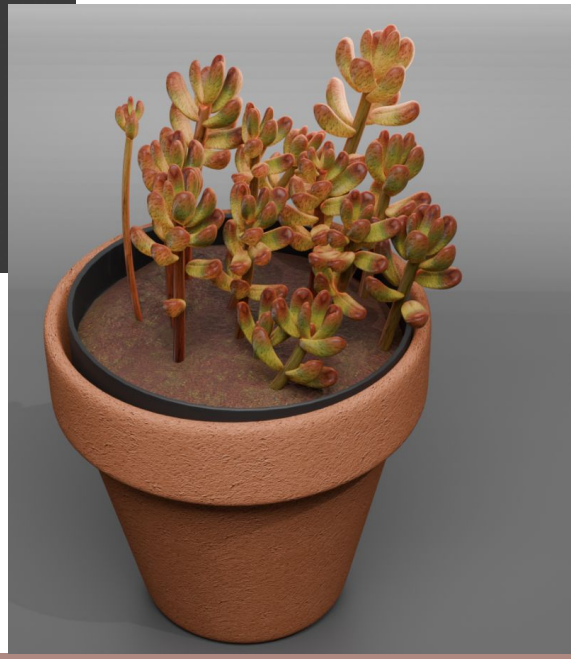
Joshua Starks

Student Reference Texture Highlights!

Steve Chia



Marcus Polson



Ji Ryoo

Student Reference Texture Highlights!

Callie Liao

Sole Mayeski

Housekeeping

- HW5 released (Last one!)
- Cat Hicks additional resources posted on Ed
- Matt Larsen guest lecture – Scientific Visualization!
- No post processing (no composite nodes)
- Project proposal grades out
  - 9.99/10 ~= 10/10
  - ^ To get your attention in case we really wanted you to see some feedback

Lecture Outline

- Motivation
- Forces + time integrators
- Fluid simulation + domain, generating a mesh
- Simulating cloth
- Simulating collisions
- Artistry in simulation, flocking

Doug James, Simulation / Interactivity

Karen Liu, Animation / Robotics

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Animation vs. Simulation

- **Animation**: create specific keyframes to snapshot a scene at various points in time, then connect them by “filling the gaps” with smooth motion

Animation vs. Simulation

- **Animation**: create specific keyframes to snapshot a scene at various points in time, then connect them by “filling the gaps” with smooth motion

- **Simulation**: use real life physics to model the behavior of natural phenomena as they evolve dynamically in time

Animation vs. Simulation

- **Animation**: create specific keyframes to snapshot a scene at various points in time, then connect them by “filling the gaps” with smooth motion

- **Simulation**: use real life physics to model the behavior of natural phenomena as they evolve dynamically in time

- Both allow us to generate **change in a scene over time** for e.g. movies and video games, scientific computing, etc
- Even for generating a still image, we might want to capture a specific frame of an animation or simulation

Motivation

Motivation

- We can use simulation to capture a specific frame of a simulation or animation

- Blender has many tools to do so
  - E.g. simulate splashing water and save the geometry from a specific frame to export

Ryan Eberhardt, 2017

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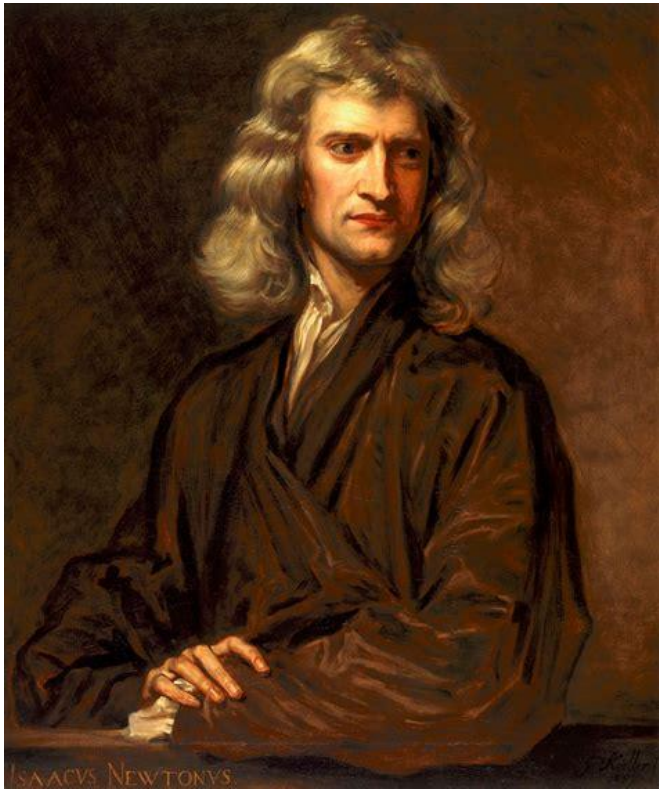
Physical Systems

- We model an object in space at some time with a **physical state**:
  - **Mass** of the object
  - **Position** of the object
  - **Velocity** of the object
  - **External forces** on the object



Physical Systems

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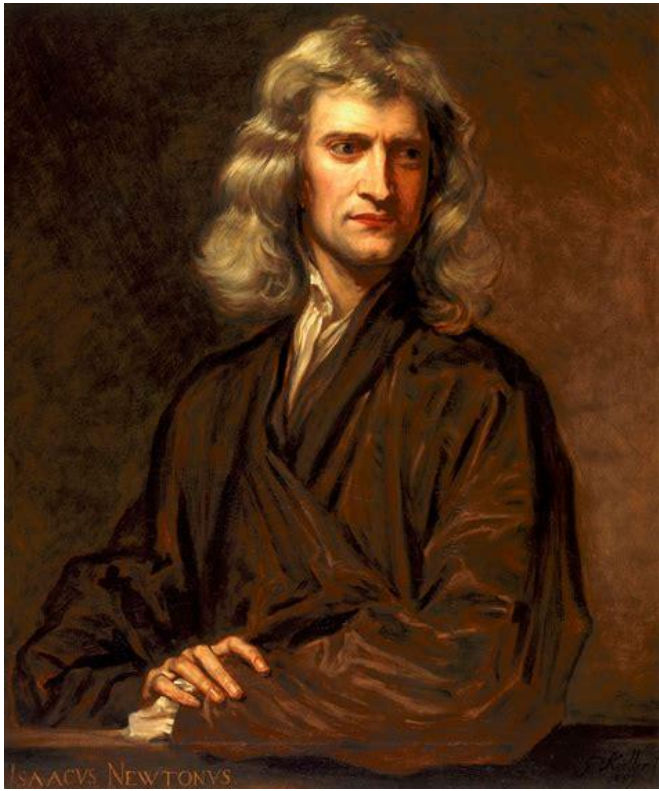
- Goal of simulation
  - Given an initial state at time *t*, compute the next state at time *t + 1*



Physical Systems

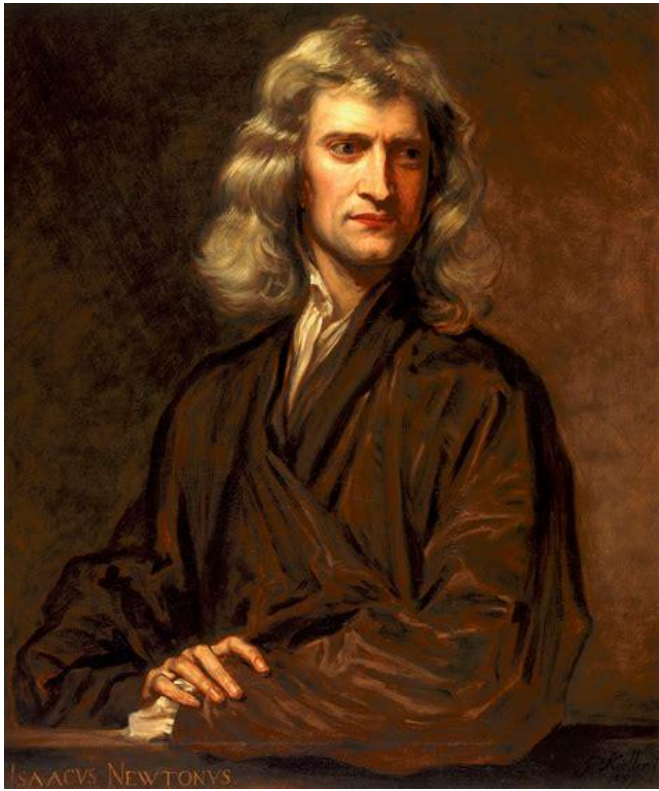
- Goal of simulation
  - Given an initial state at time *t*, compute the next state at time *t + 1*

- Newton's 2nd Law of Physics:

$$F = ma$$

- Applying a force on an object of some mass will cause it to move with some acceleration

$$a = F/m$$

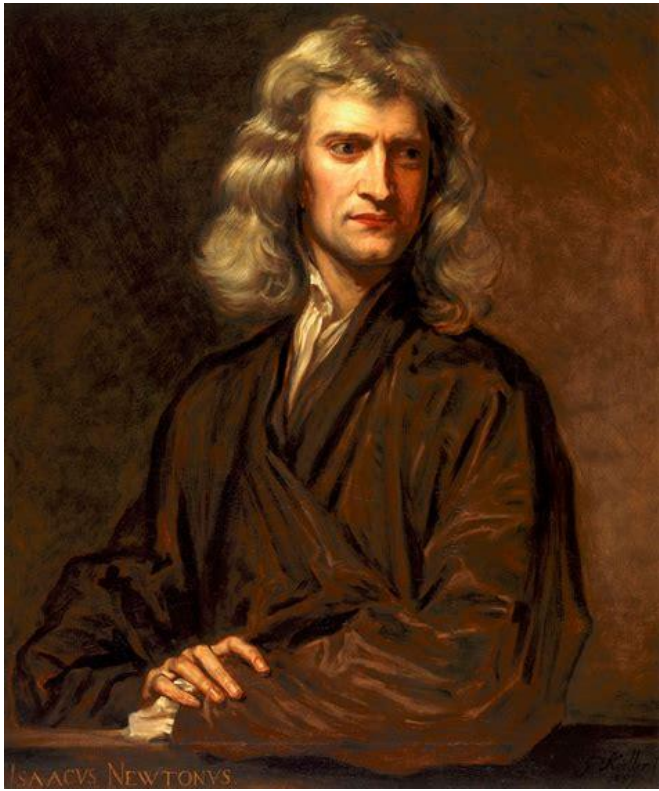


Physical Systems

- Newton's 2nd Law of Physics:

$$F = ma$$

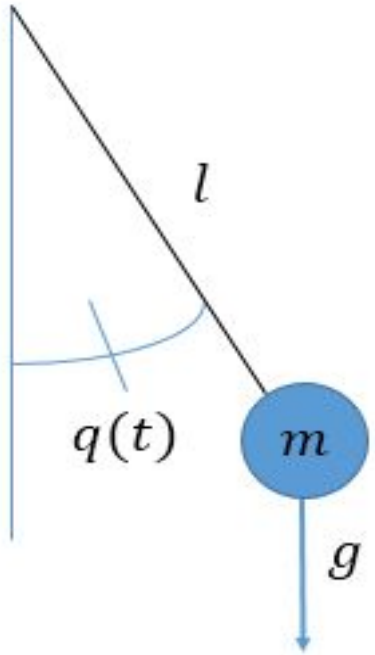
- Types of forces:
  - Constant forces (e.g. gravity)
  - Time dependent forces (e.g. wind)
  - Position dependent forces (e.g. magnets)
  - Velocity dependent forces (e.g. friction)
  - Position and velocity dependent forces (e.g. springs)



Simple Example: The Pendulum

- Physical state of the pendulum at time t
  - Mass of the object → **m**
  - Position of the object → **q(t)**
  - Velocity of the object → **v(t)**
  - External forces on the object → **a(t)**
    - The force of gravity generates an acceleration a(t)

- We can define the state of our system depending on the application
  - E.g. using angle for position



Simple Example: The Pendulum

- Using our state variables, write our **equations of motion** from t to t + **time step** Δt:

$$a(t) = \ddot{q} = \frac{d^2q}{dt^2} = -\frac{g}{l}\sin q$$

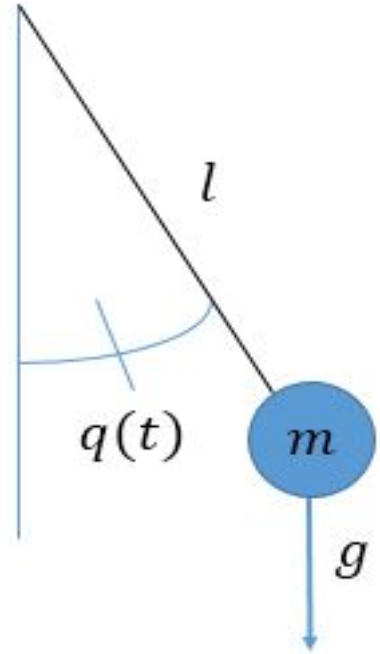
(derived from physics using force of gravity)

$$v(t + \Delta t) = v(t) + (\Delta t)a(t)$$

(velocity final = velocity initial + a * t)

$$q(t + \Delta t) = q(t) + (\Delta t)v(t)$$

(position final = position initial + v * t)



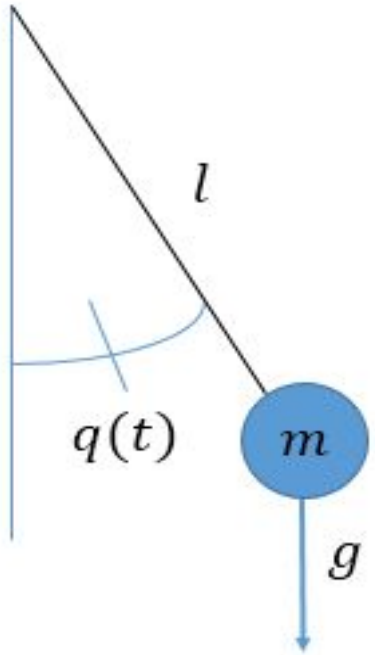
Simple Example: The Pendulum

- This **numerical method** for **updating** our **state** variables from state k to k+1 is called **Explicit Euler** (Forward Euler):

$$v_{k+1} = v_k - (\Delta t)\frac{g}{l}\sin q_k$$

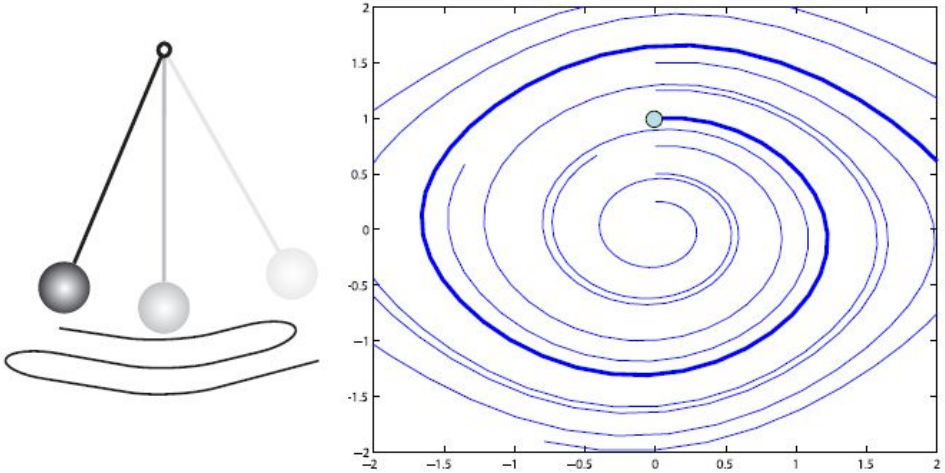
$$q_{k+1} = q_k + (\Delta t)v_k$$

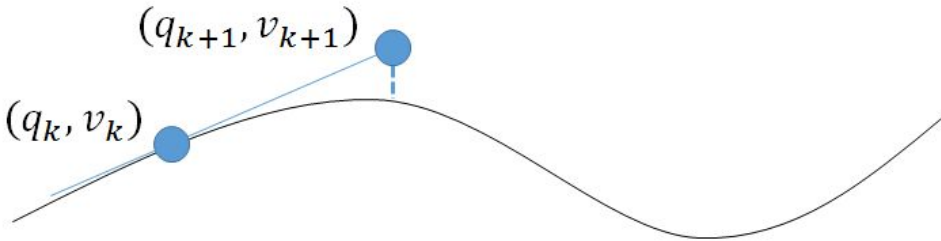
- Explicit Euler one of many “time integrators”



Time Integrators

- Not all time integrators are stable
- For Explicit Euler, might have to increase the amplitude of the pendulum every update
- Our equations are approximations (1st order Taylor series)





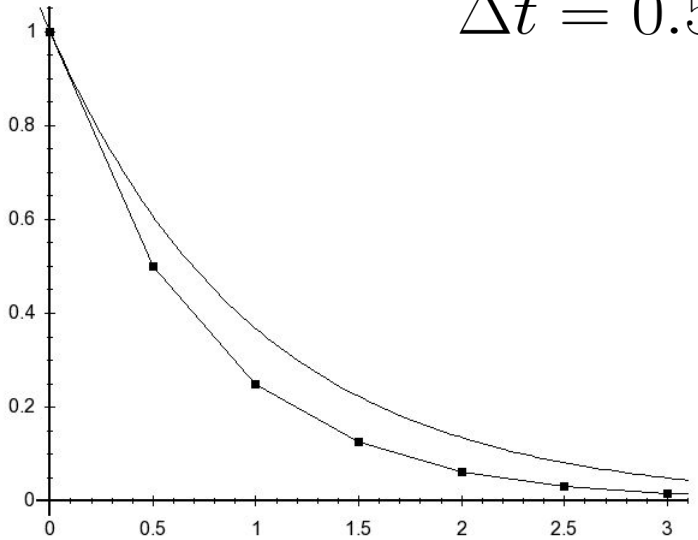
$$q_{k+1} = q_k + (\Delta t) v_k$$

Time Integrators

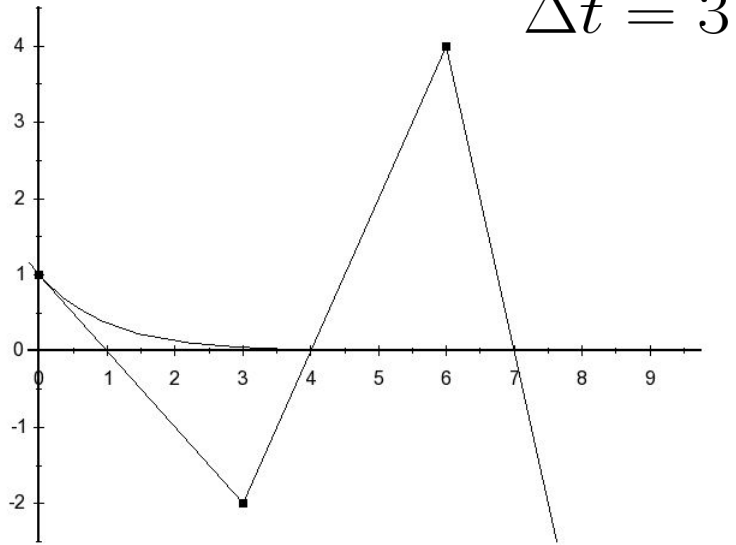
- The key to stability is in the **time step** Δt

- Example: Forward Euler on the equation $v(t) = -q(t)$

$\Delta t = 0.5$



$\Delta t = 3$



Time Integrators

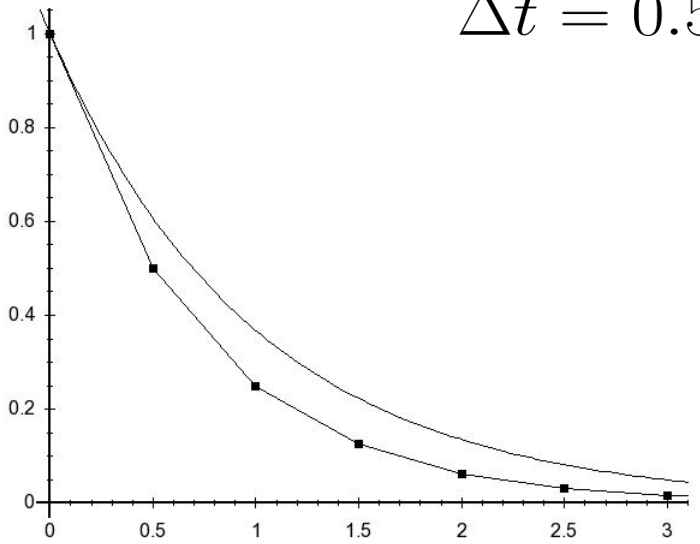
- The key to stability is in the **time step** Δt
- Example: Forward Euler on the equation $v(t) = -q(t)$

$$q_{k+1} = q_k + (\Delta t)v_k$$
$$= q_k - (\Delta t)q_k$$
$$= (1 - \Delta t)q_k$$

- Exponential decay when:

$$|1 - \Delta t| < 1$$
$$-2 < -\Delta t < 0$$

$\Delta t = 0.5$



Time Integrators

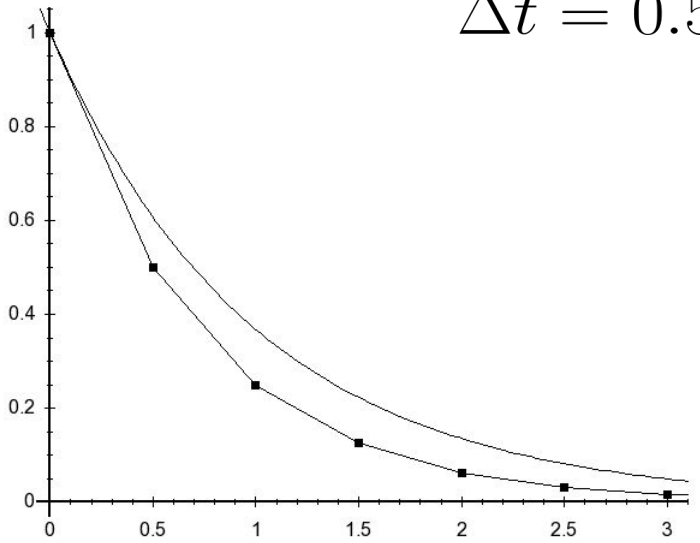
- The key to stability is in the **time step** Δt
- Example: Forward Euler on the equation $v(t) = -q(t)$
  Stable when:

$$|1 - \Delta t| < 1$$

$$-2 < -\Delta t < 0$$

- Can do similar analyses for other applications of Explicit Euler to find the **time step restriction**
- Some applications use **adaptive time steps** that change over time

$\Delta t = 0.5$

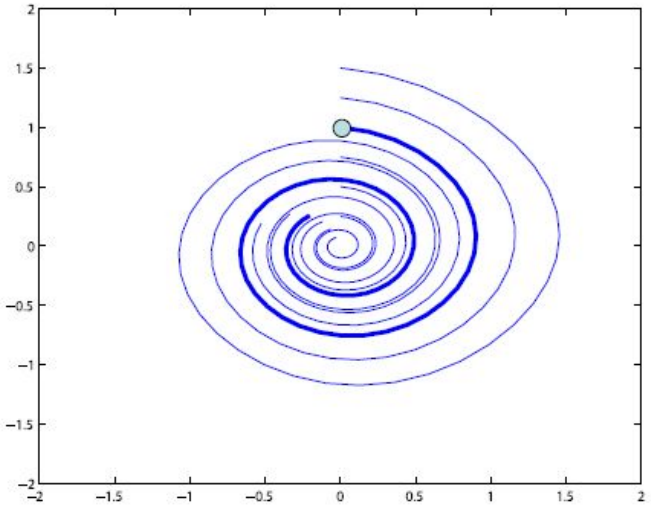


Other Time Integrators

- There are many time integrators besides Explicit Euler, and some are still being developed for various applications!
- Consider **Implicit Euler** (aka Backward Euler):

$$q_{k+1} = q_k + (\Delta t) v_{k+1}$$

$$v_{k+1} = v_k + (\Delta t) a_{k+1}$$



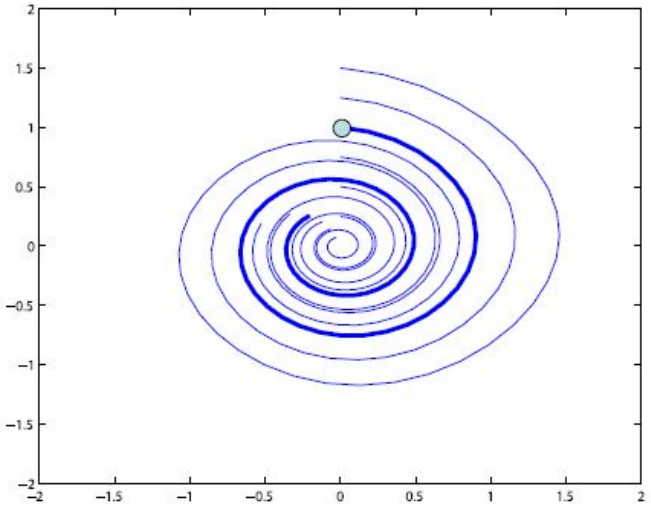
Other Time Integrators

- There are many time integrators besides Explicit Euler, and some are still being developed for various applications!
- Consider **Implicit Euler** (aka Backward Euler):

$$q_{k+1} = q_k + (\Delta t) v_{k+1}$$

$$v_{k+1} = v_k + (\Delta t) a_{k+1}$$

- Can do analysis to see this is stable for ALL time steps!
- But requires implicit solve and may cause damping



Other Time Integrators

- The Runge-Kutta methods are a series of popular time integrators

- Intuition with Runge-Kutta 2 (RK2):
  Take 2 Forward Euler steps:

$$q_{k+1} = q_k + (\Delta t) v_k$$

$$q_{k+2} = q_{k+1} + (\Delta t) v_{k+1}$$

  Then average:

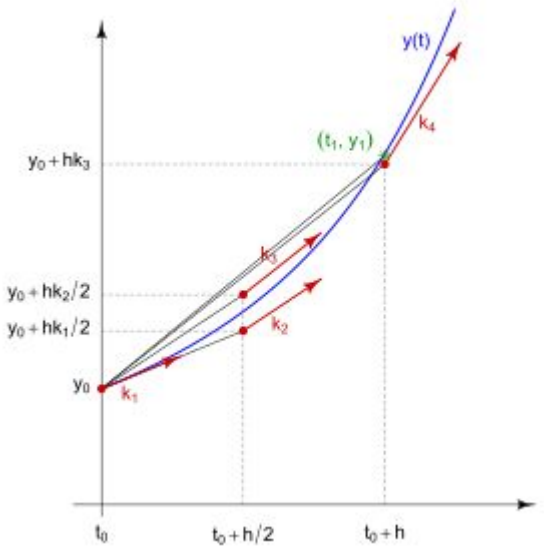
$$q_{k+1} = \frac{1}{2}(q_k + q_{k+2})$$

Other Time Integrators

- The Runge-Kutta methods are a series of popular time integrators
- Most popular is Runge-Kutta 4 (RK4): Weighted-average of 4 steps:

$$y_{n+1} = y_n + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)h,$$
$$t_{n+1} = t_n + h$$

$$\begin{aligned} k_1 &= f(t_n, y_n), \\ k_2 &= f\left(t_n + \frac{h}{2}, y_n + h\frac{k_1}{2}\right), \\ k_3 &= f\left(t_n + \frac{h}{2}, y_n + h\frac{k_2}{2}\right), \\ k_4 &= f(t_n + h, y_n + hk_3). \end{aligned}$$

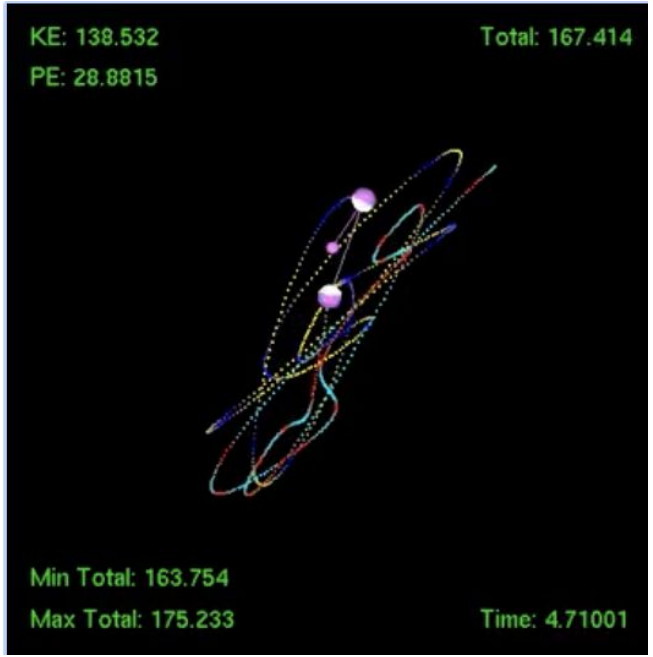


Pendulum Simulation in Action

https://drive.google.com/file/d/1CuD_AHgjv5KBP7J-6R-MMCuVstm7Q-DR/view?usp=drive_link

Pendulum Simulation in Action

- PE + KE stays constant in perfect world
- But our computers have numerical imprecision
- Many schemes like explicit/implicit Euler have different ways of handling the error



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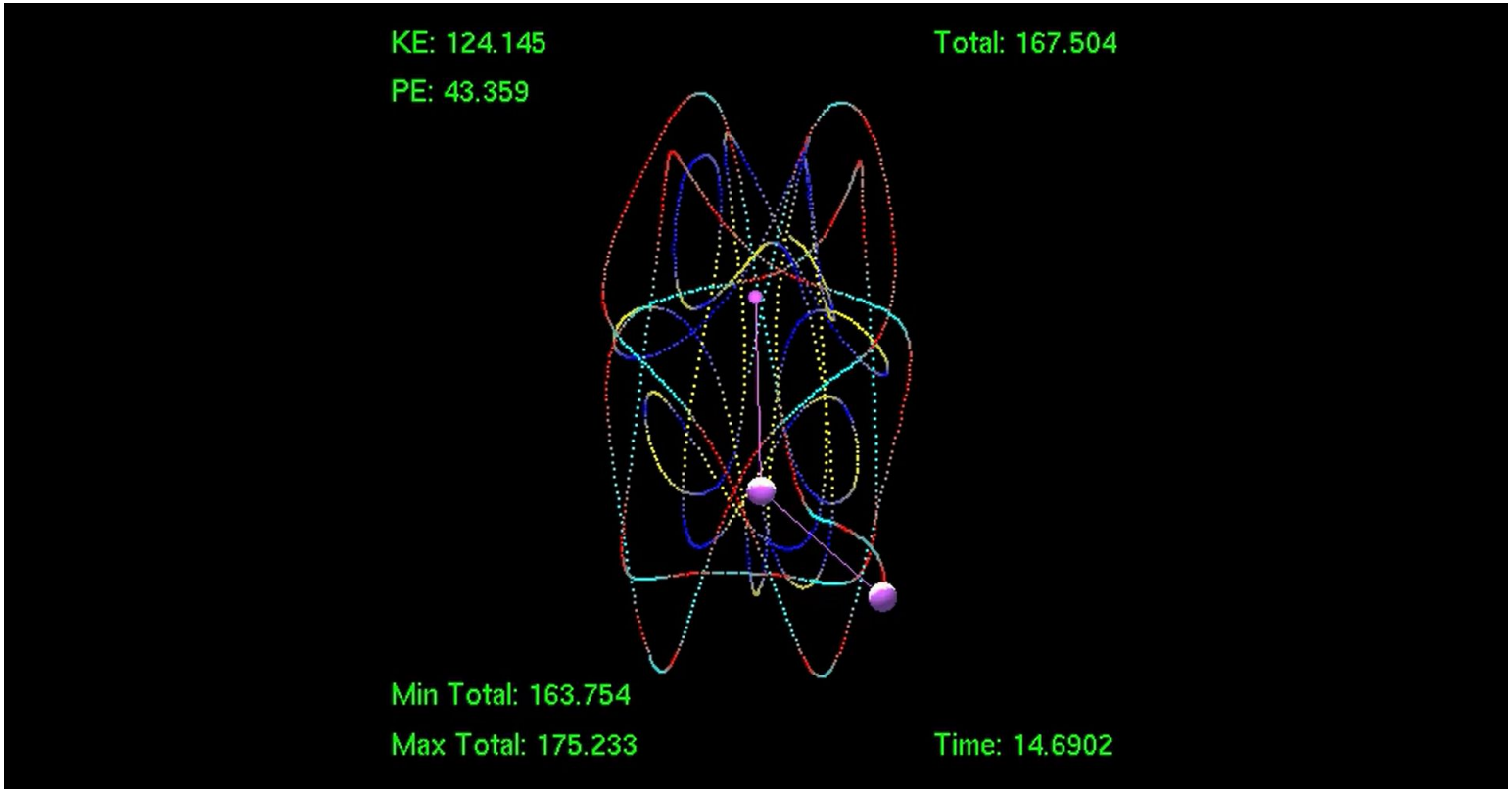

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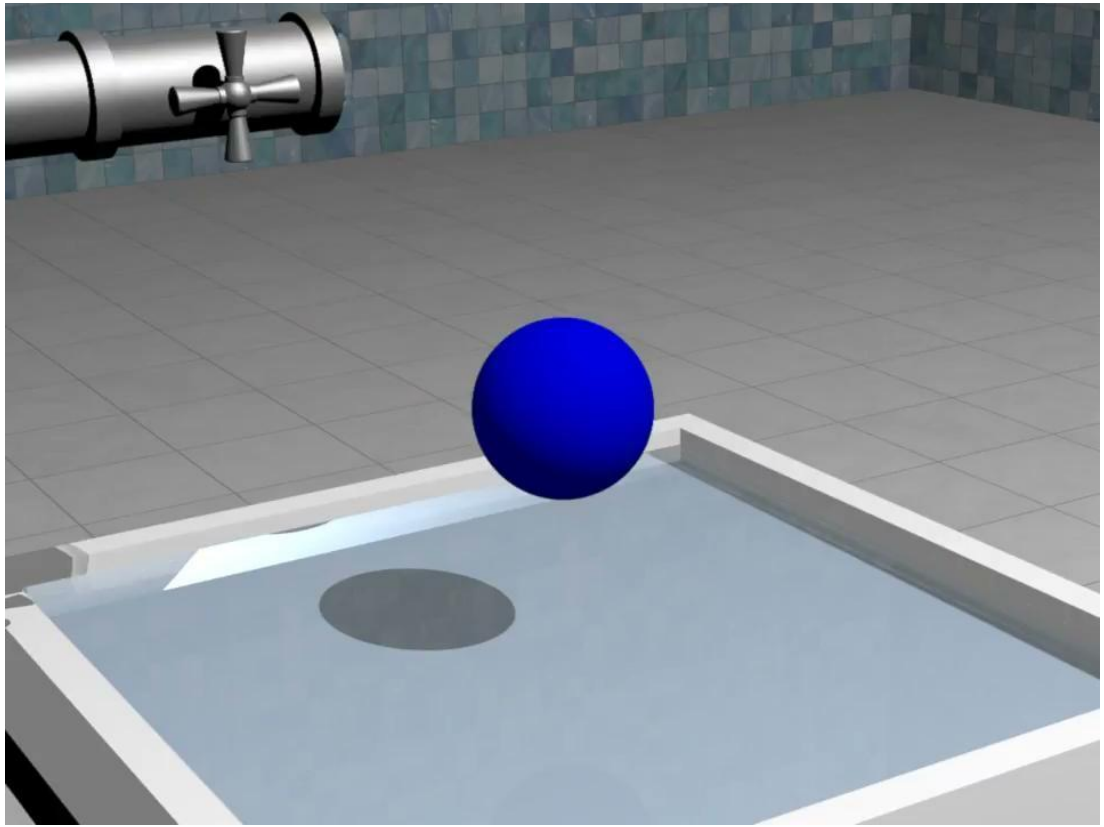
Fluid Simulation



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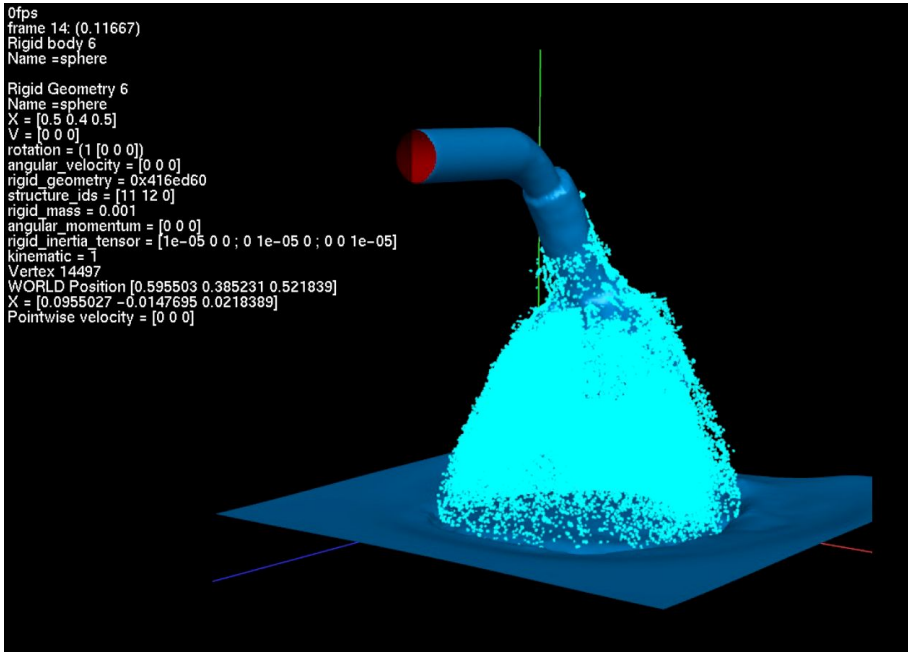
Fluid Simulation

- Refers to the simulation of liquids and gases as they move around in some space over time due to some force(s)

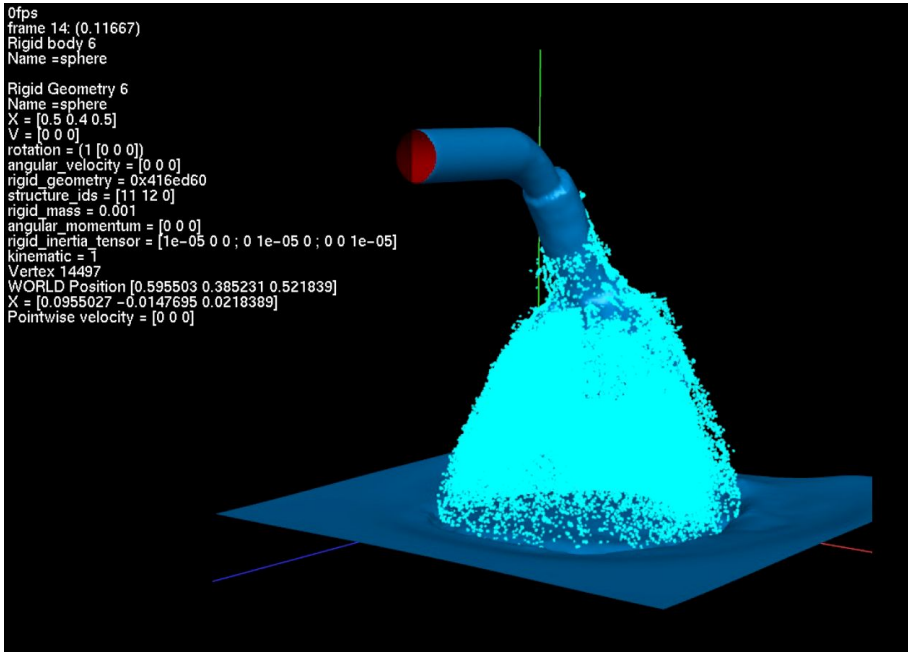
- A huge research space
- Many methods have been developed



Fluid Simulation

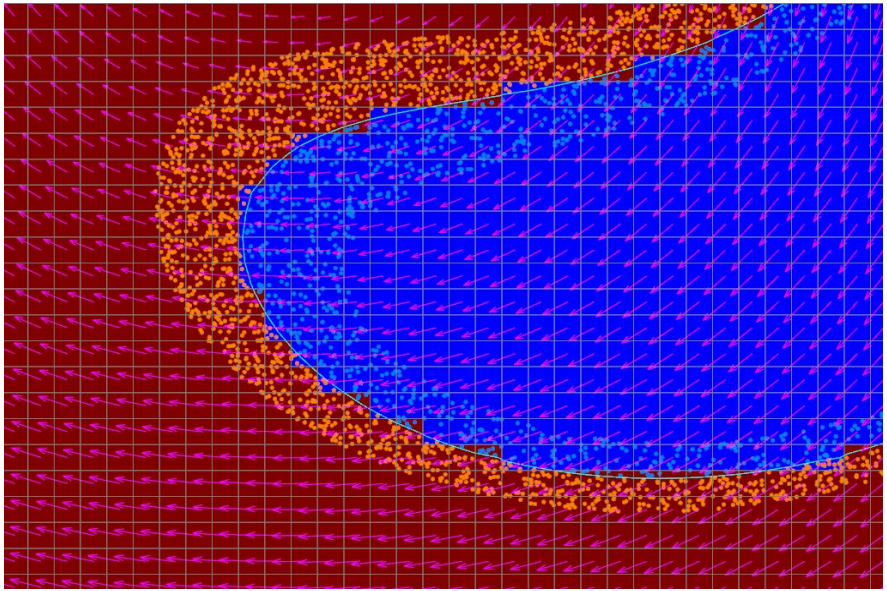
- **Lagrangian fluid simulation** tracks lots of **particles** as they move through space and time

- Each particle has its own physical state at time t of:
  - Mass
  - Position
  - Velocity
  - Force / Acceleration



Fluid Domain

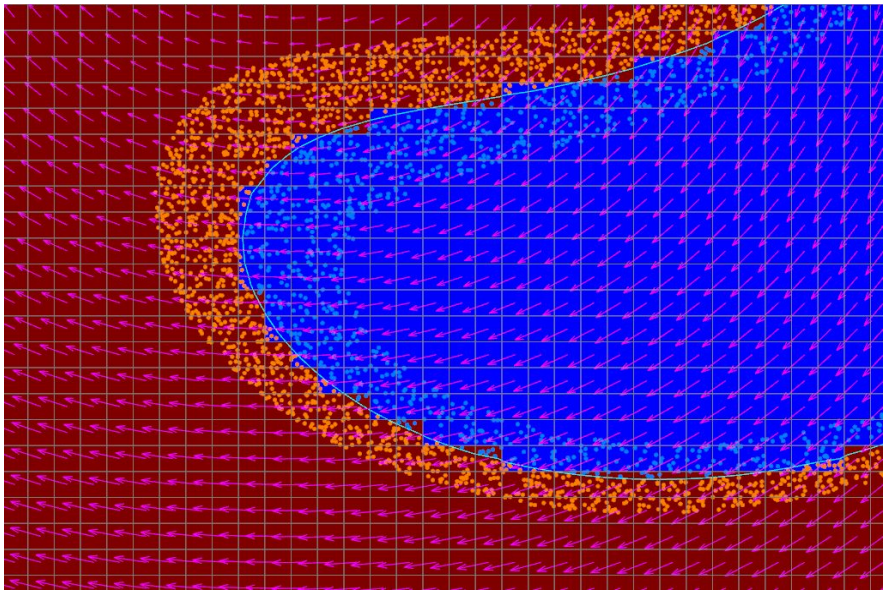
- We track the particles in space using a **fluid domain**
- In 2D we use a rectangular grid, in 3D we use a bounding volume



Fluid Domain

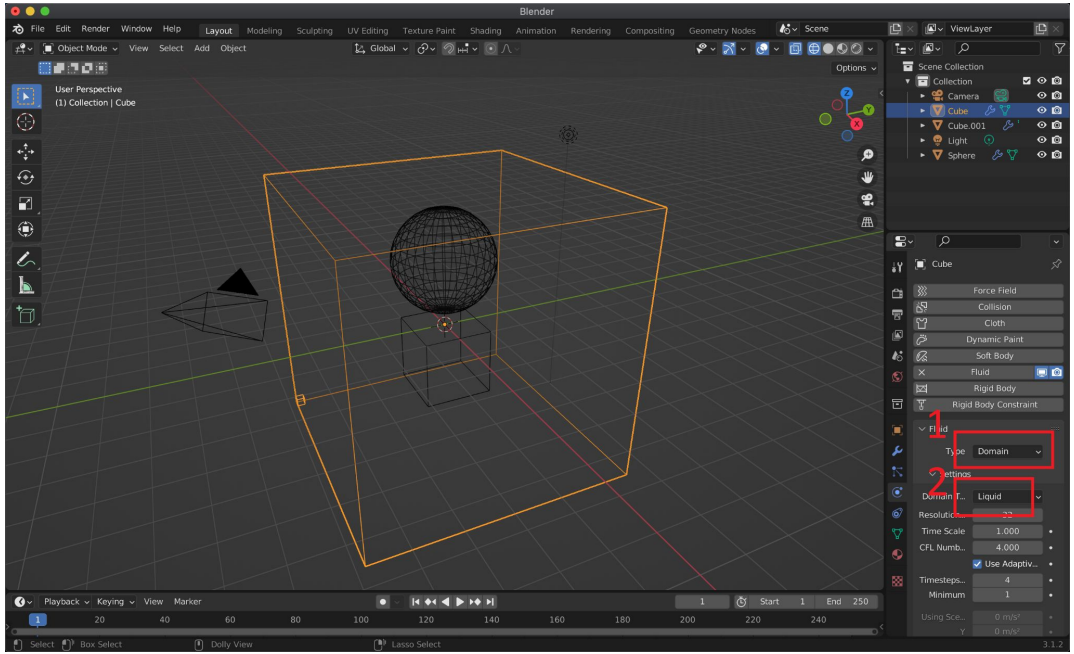
- We track the particles in space using a **fluid domain**
- In 2D we use a rectangular grid, in 3D we use a bounding volume

- 2D Example:
  - Each **grid cell** stores **velocity vectors** along edges
  - Essentially, the domain stores a **velocity field**
  - These velocities are used to move/**advect** the particles in the cell to the next state



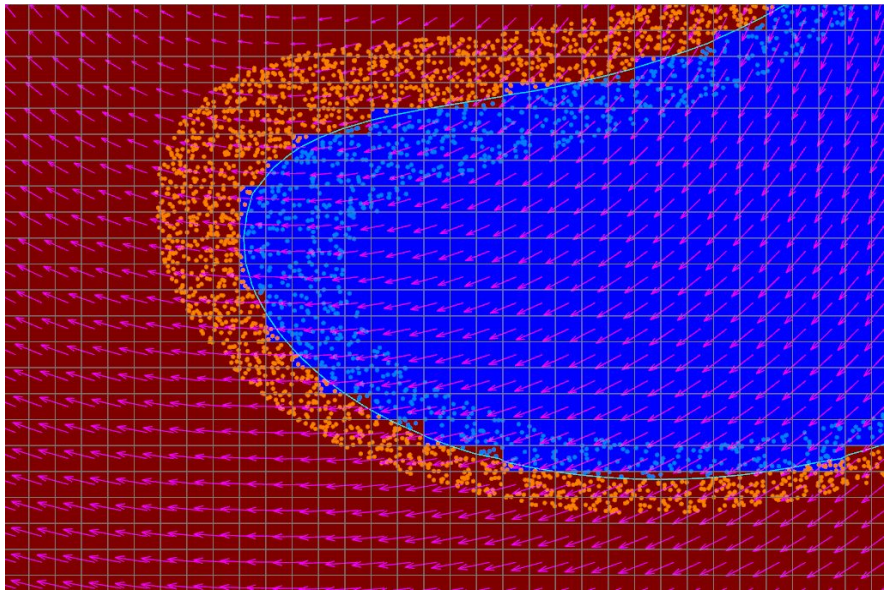
Fluid Domain

- We track the particles in space using a **fluid domain**
- Blender fluid domain with a gravity force field



Eulerian Fluid Domain

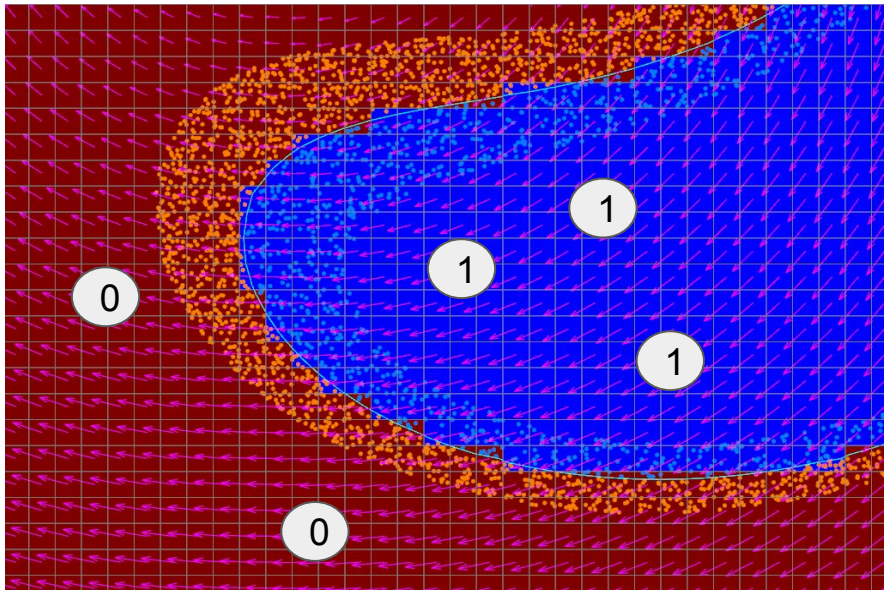
- We sometimes also represent the fluid with the fluid domain itself by marking grid cells as “on” or “off” if they’re in the fluid of interest



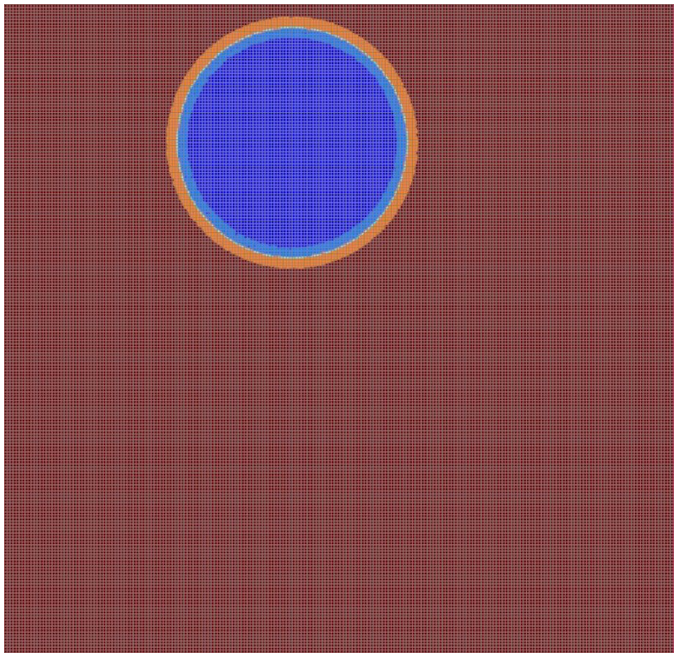
Eulerian Fluid Domain

- We sometimes also represent the fluid with the fluid domain itself by marking grid cells as “on” or “off” if they’re in the fluid of interest

- **Eulerian fluid simulation** tracks the fluid with the **grid**
- In the example –
  blue grid cell = water
  red grid cell = air
- Check if a grid cell is “inside” the water domain and mark the cell blue if so

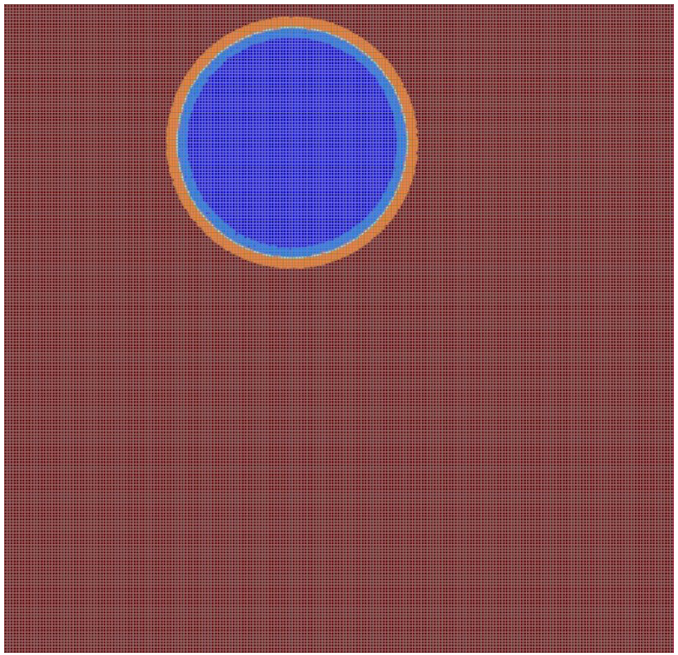


- How do we know if we're "inside" the fluid we care about?



Implicit Surfaces

- How do we know if we're "inside" the fluid we care about?
- Recall **implicit surfaces**, e.g. when we ray traced implicit spheres



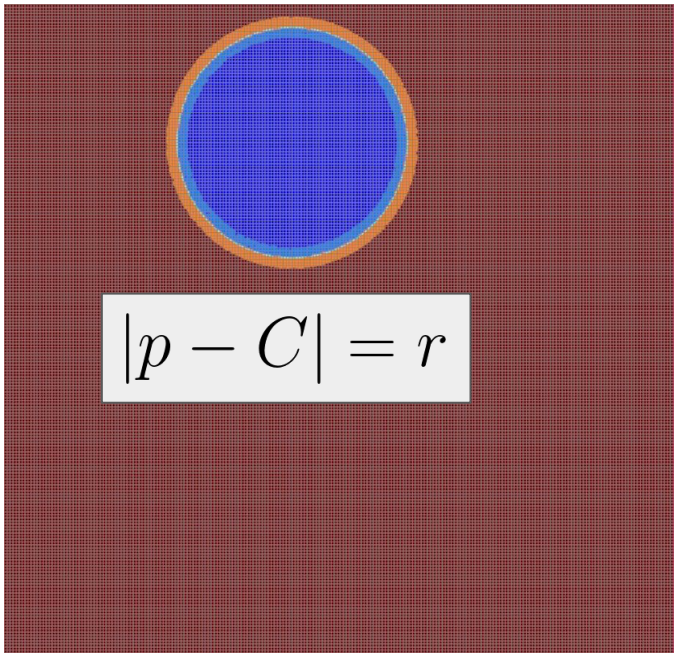
Implicit Surfaces

- How do we know if we're "inside" the fluid we care about?
- Recall **implicit surfaces**, e.g. when we ray traced implicit spheres

- Consider starting with a very simply shaped fluid, e.g. a circle of water
- Can define the **implicit function** as a **signed distance function**:

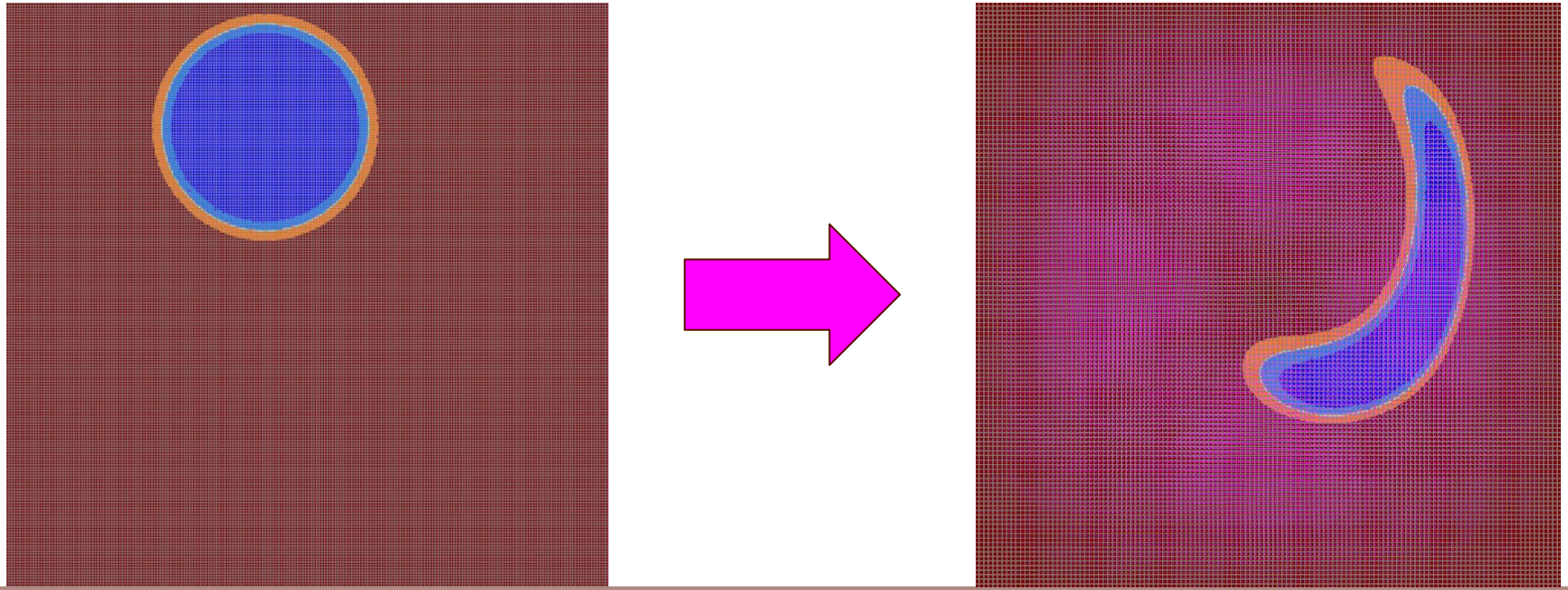
$$\phi(p) = \sqrt{(p - C) \cdot (p - C)} - r$$

- If = 0, then on boundary of water/air
- If < 0, then inside circle (is in water)
- If > 0, then outside circle (is in air)



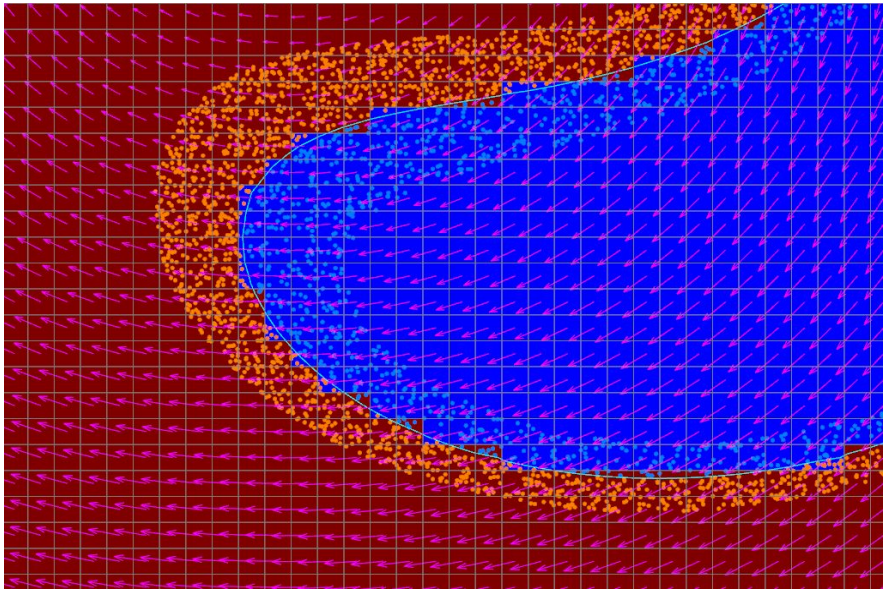
Implicit Surfaces

- Storing the **implicit function** values on each grid cell turns the grid into a **signed distance field** that can move with the **velocity field**



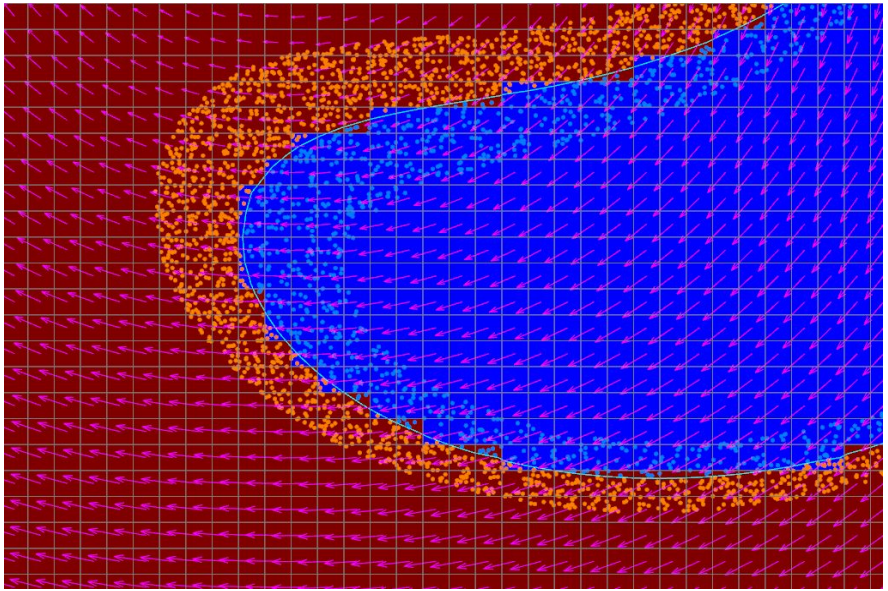
Mix of Lagrangian & Eulerian

- How could we compromise for efficiency and performance?



Mix of Lagrangian & Eulerian

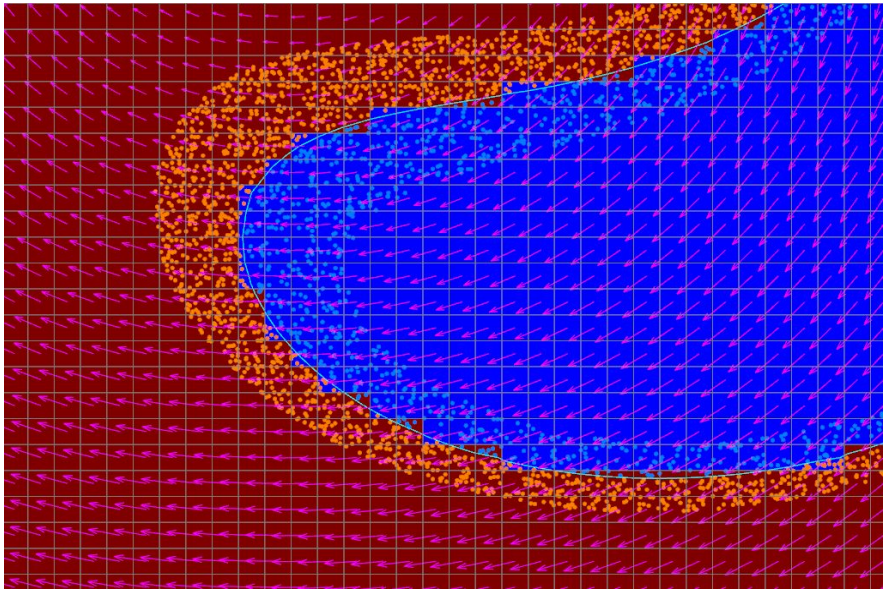
- Can use **Lagrangian marker particles** for the **fluid boundary**, then use an **Eulerian grid** to represent the rest of the fluid domain



Mix of Lagrangian & Eulerian

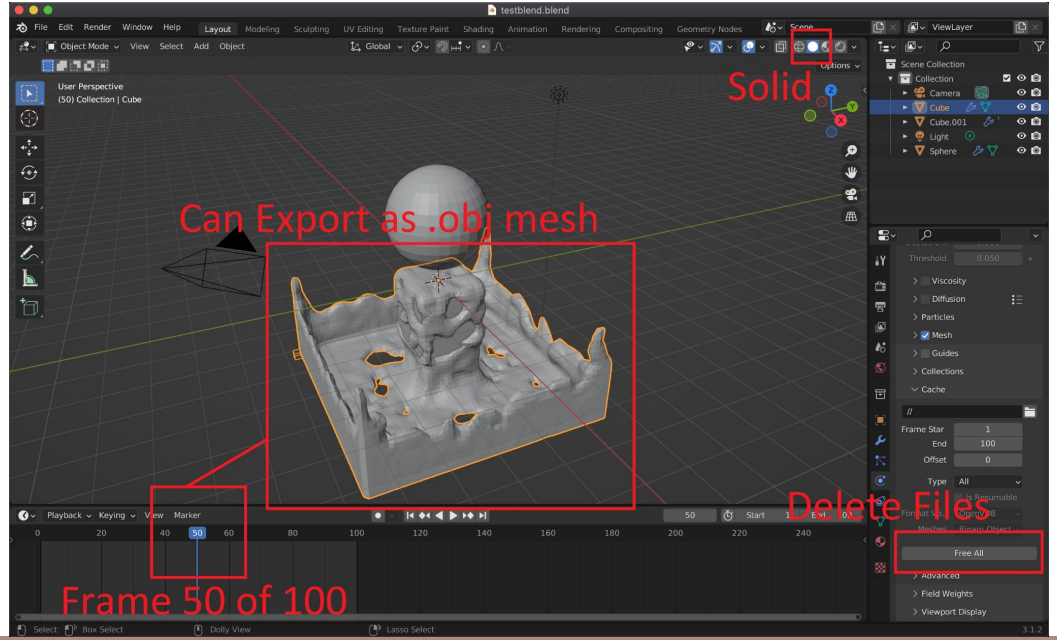
- Can use **Lagrangian marker particles** for the **fluid boundary**, then use an **Eulerian grid** to represent the rest of the fluid domain

- Orange and light blue marker particles track the boundary between water and air
- Only need to store in memory a “few” marker particles, then just use an inexpensive grid to track the “inside” of the water



Generating a Mesh

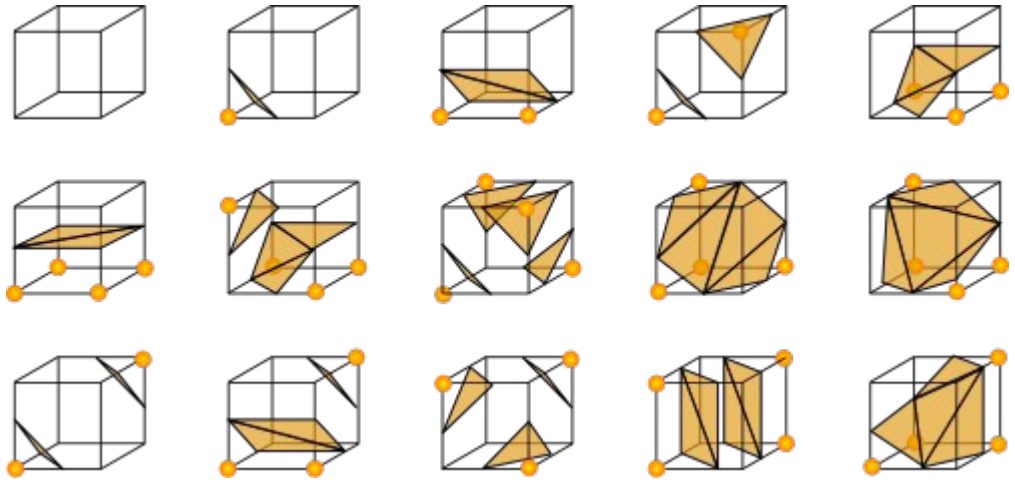
- After doing our fluid simulation, we want to turn it into a mesh (e.g. many triangles)



Generating a Mesh

- After doing our fluid simulation, we want to turn it into a mesh (e.g. many triangles)

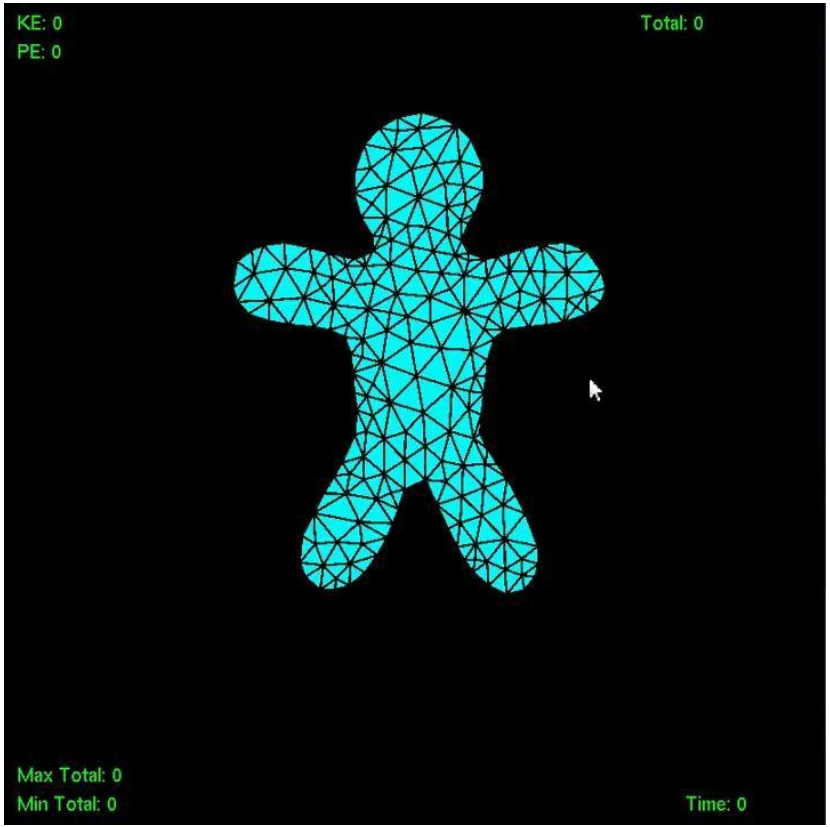
- One such algorithm is **marching cubes**
- Check each grid cell of the fluid domain
- 2^8 = 256 configurations of polygons based on which corners of the cell are “inside” e.g. water



Lecture Outline

- Motivation
- Forces + time integrators
- Fluid simulation + domain, generating a mesh
- Simulating cloth
- Simulating collisions
- Artistry in simulation, flocking

Elasticity



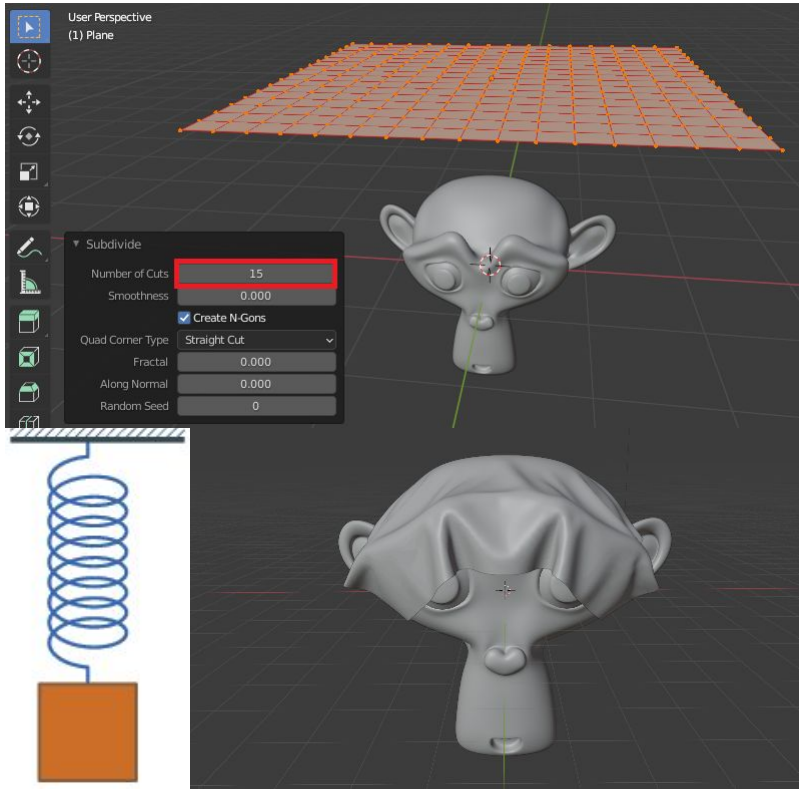
https://drive.google.com/file/d/1QW-xRso_LhAtCgOBVhWDr302dJ0QUzda/view?usp=drive_link

Elasticity for Cloth Simulation

- **Spring forces** are position and velocity dependent forces of the form:

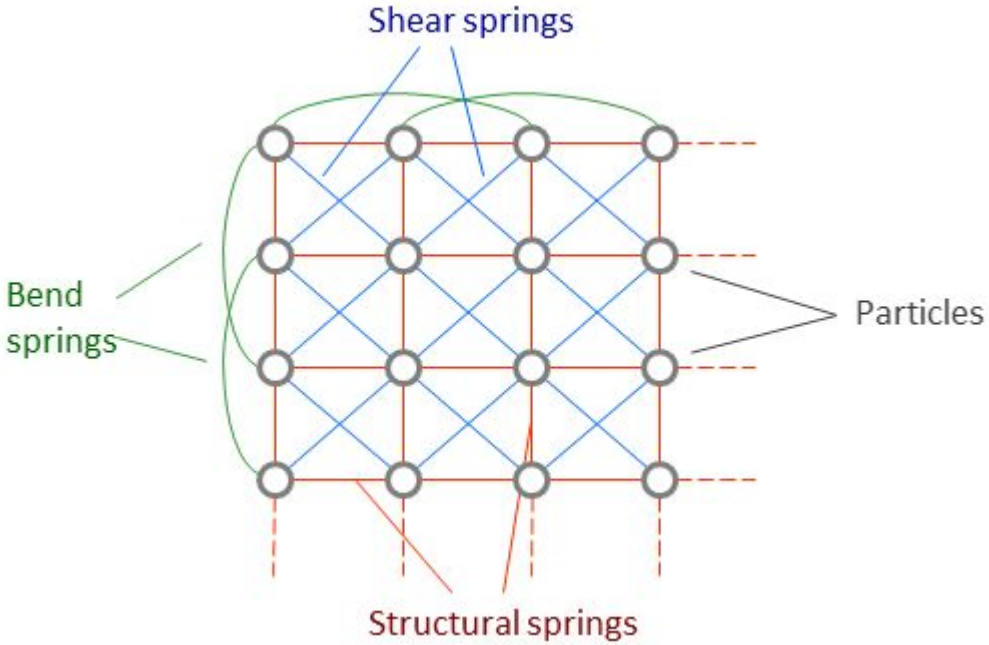
$$F_{spring} = -kq(t) - k_d v(t)$$

- Simplest **cloth simulation** models the cloth as a grid of **particles** that connect via a **network of springs**
- Also still ongoing research!



Elasticity for Cloth Simulation

- **Structural springs**
  connect particles
- **Shear springs**
  resist shear motion
- **Bend / Flexion springs**
  resist out-of-plane motion



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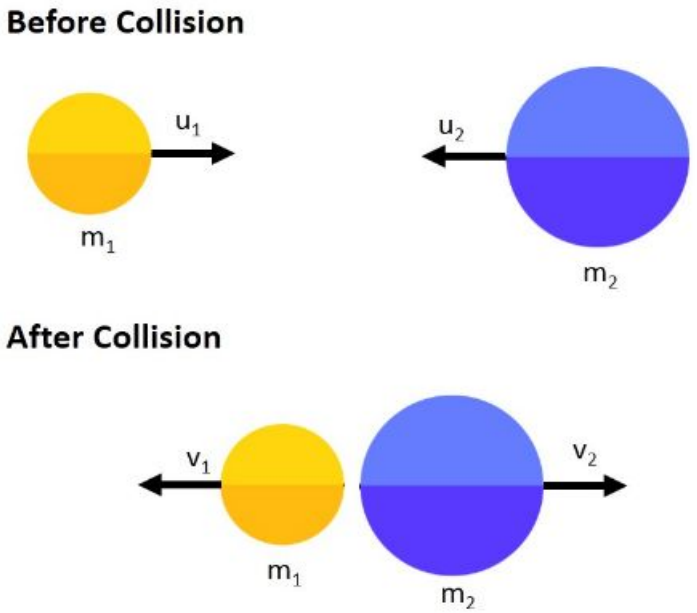
Simulating Collisions: 1D

- Suppose 2 particles **a** & **b** each with their own mass and velocity collide
- Newton's 3rd law says every action has an equal and opposite reaction, i.e. the force on object **a** equals the force on object **b**

- Leads to the idea of **conservation of momentum**

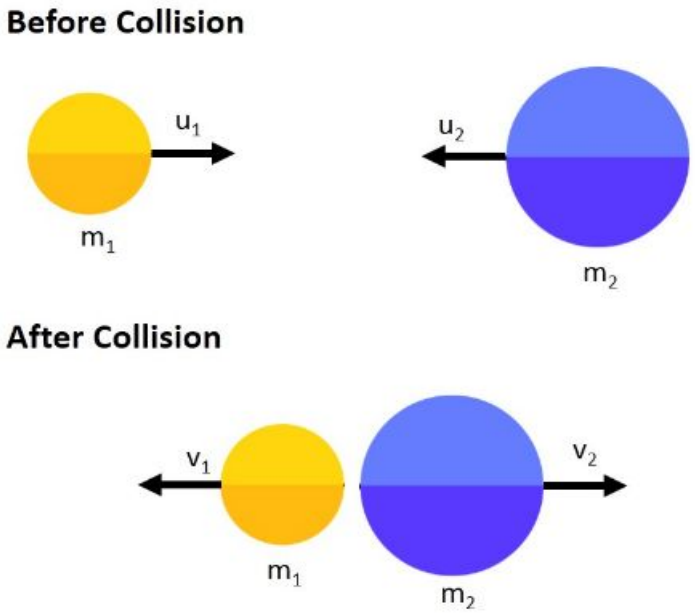


$$m_a u_a + m_b u_b = m_a v_a + m_b v_b$$

Simulating Collisions: 1D

- Define the **coefficient of restitution**

$$c_R = -\frac{v_b - v_a}{u_b - u_a}$$



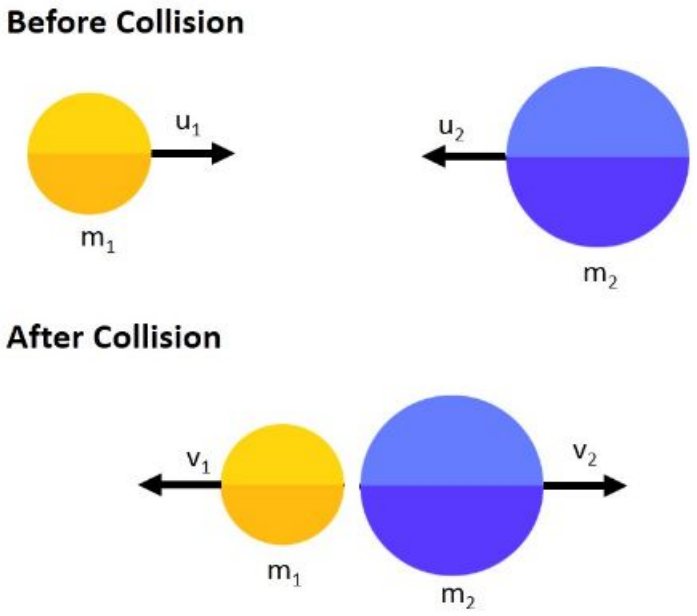
$$m_a u_a + m_b u_b = m_a v_a + m_b v_b$$

Simulating Collisions: 1D

- Define the **coefficient of restitution**

$$c_R = -\frac{v_b - v_a}{u_b - u_a}$$

- If 0, then objects stick together after collision, i.e. **inelastic collision**
- If 1, then objects bounce off each other without losing any energy, i.e. completely **elastic collision**
- If between 0 and 1, then some of the **energy is lost to heat, etc**



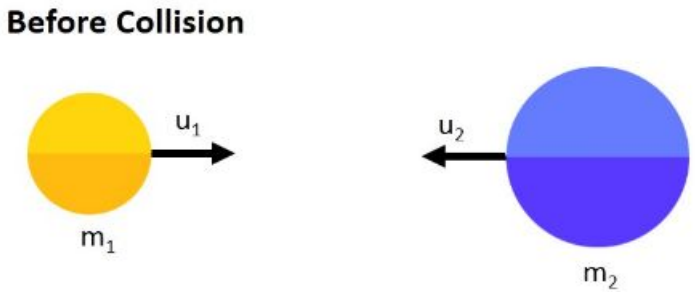
$$m_a u_a + m_b u_b = m_a v_a + m_b v_b$$

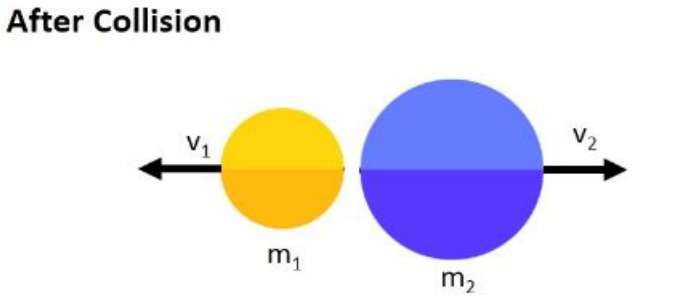
Simulating Collisions: 1D

- Set the **coefficient of restitution** to some desired model value:

$$c_R = -\frac{v_b - v_a}{u_b - u_a}$$

- We know the velocities before the collision
- Use equation for cR and equation for conservation of momentum to **solve for the new velocities** after collision





$$m_a u_a + m_b u_b = m_a v_a + m_b v_b$$

Simulating 3D Collisions: Point-Plane

- Remember most of our geometry will be triangles, so most collisions will be **point-plane intersections**

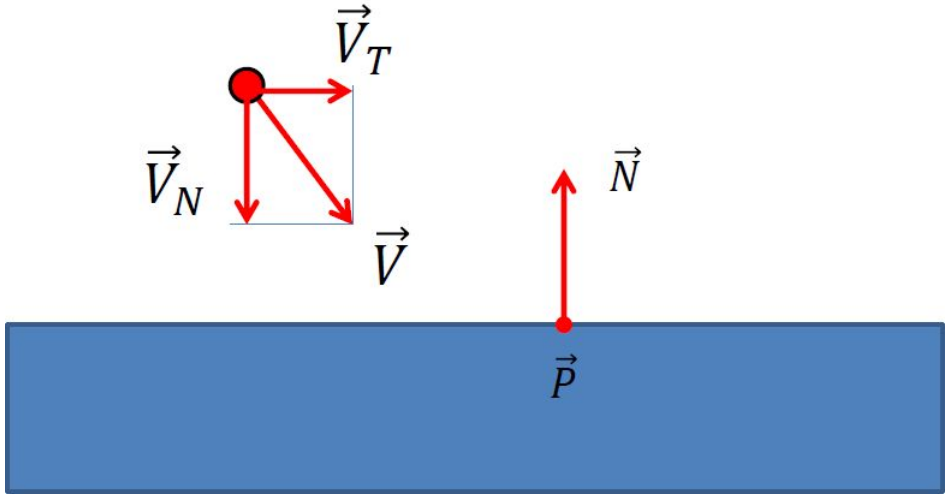
Simulating 3D Collisions: Point-Plane

- Remember most of our geometry will be triangles, so most collisions will be **point-plane intersections**

- Reuse ideas from ray-plane intersections!
  - A particle's initial position is like the origin of a ray
  - A particle's velocity is like the direction of a ray

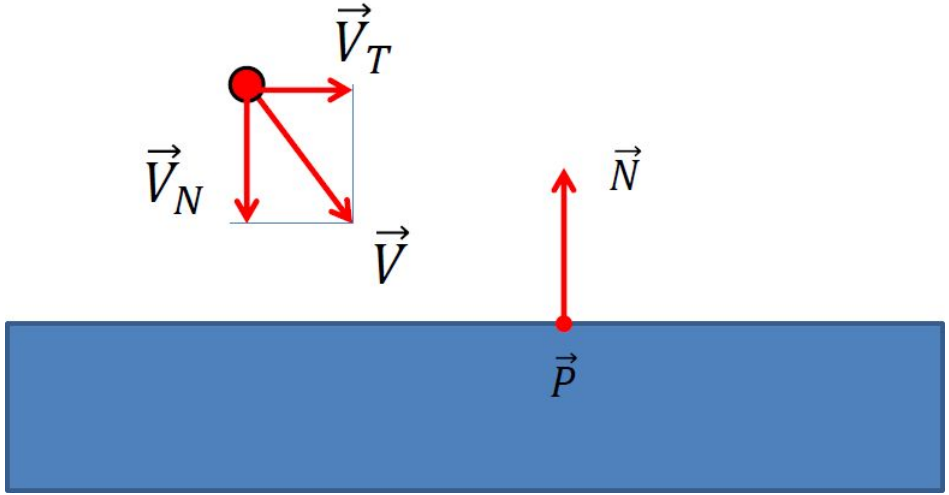
Simulating Collisions: Point-Plane

- Note: collisions with planes only involve the **normal component** of the velocity



Simulating Collisions: Point-Plane

- Note: collisions with planes only involve the **normal component** of the velocity
- This simplifies point-plane collisions in 3D into a 1D collision problem!
- Solve using just the normal components of velocities



- **Tangential components** stay the same unless **friction** is involved

Lecture Outline

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- Simulating collisions
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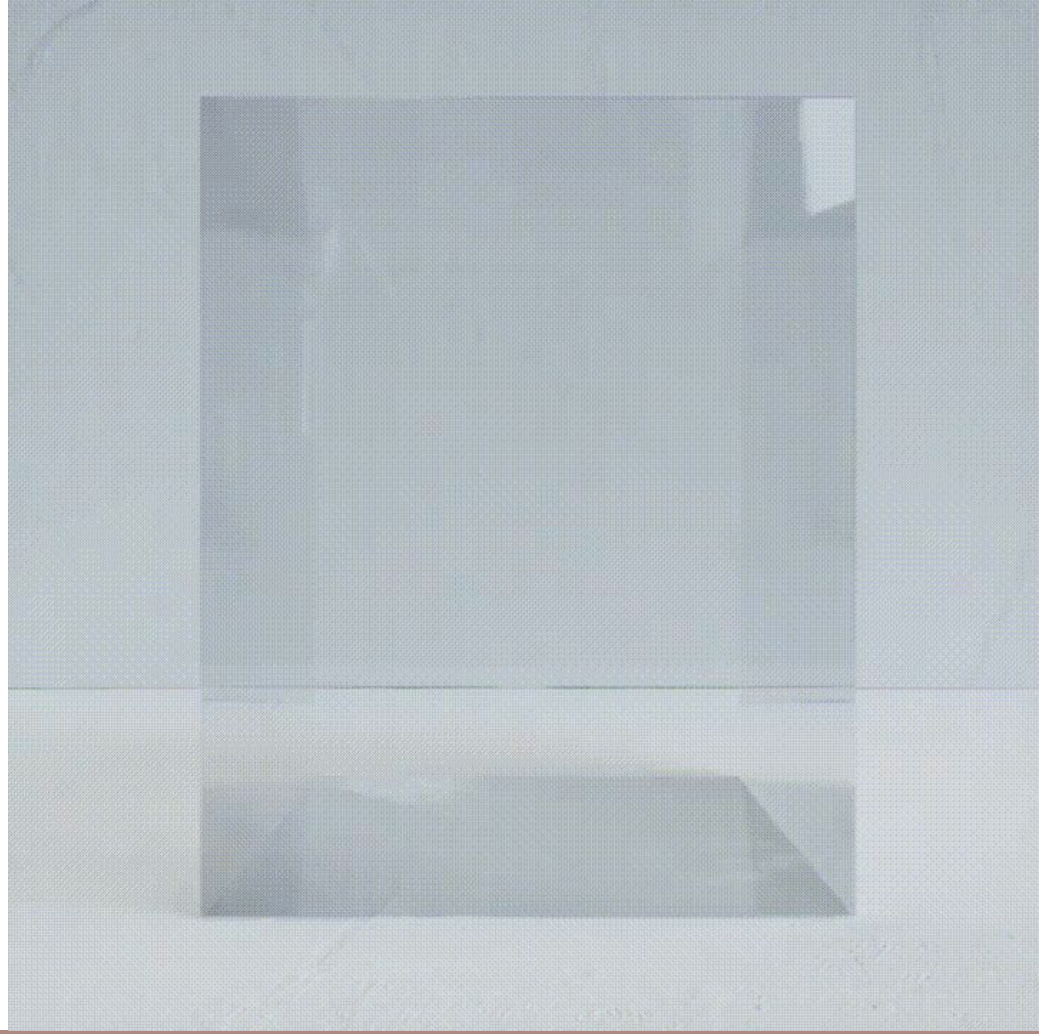
Artistry in Simulation

- You can simulate anything… including things that defy physics!

Artistry in Simulation

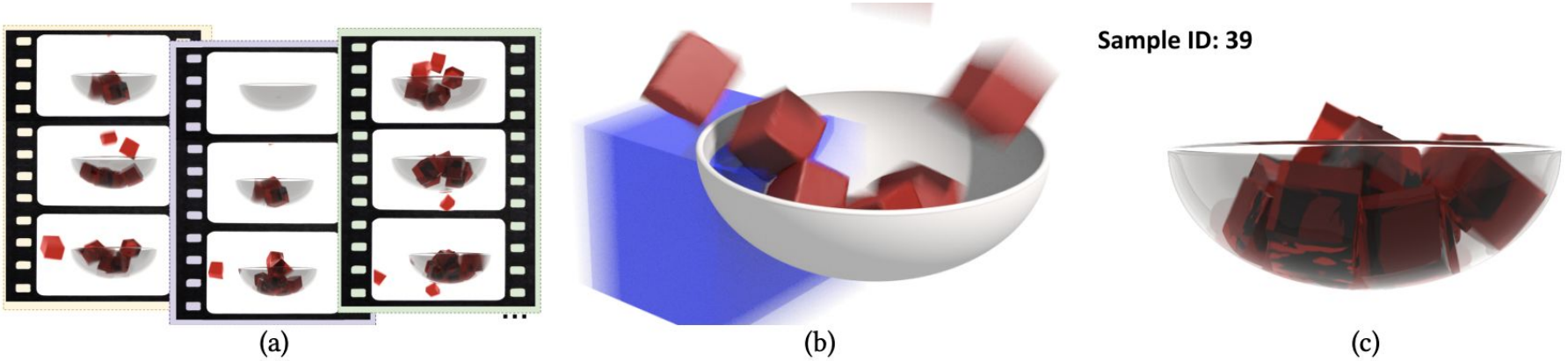
Esteban Diacono



Artistry in Simulation

Artistry in Simulation

- Or you can pick the desired outcome from many simulations



Purvi Goel, Unified Many-Worlds Browsing of Arbitrary Physics-based Animations, SIGGRAPH

Artistry in Simulation

- Or you can simulate old movie effects

The Shining, 1980

Artistry in Simulation

- Or you can simulate old movie effects

The Shining, 1980

Ready Player One, 2018

Artistry in Simulation

Spiderman: Into the Spiderverse, 2018

Flocking Simulation

What do birds do?

Flocking Simulation

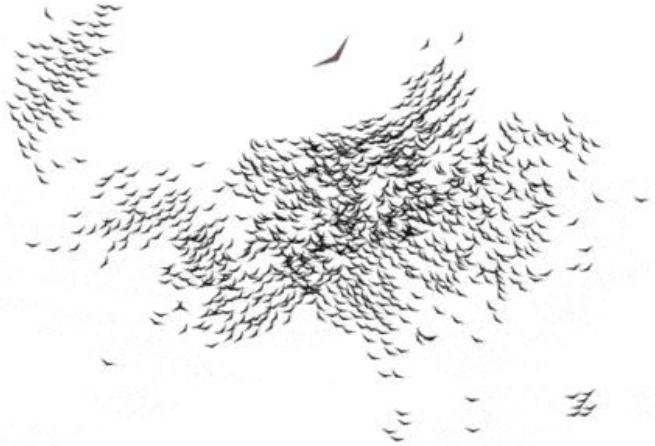
What do birds do?

- Birds like to follow each other
- Birds don't want to be too close to each other
- When flying, birds look for spaces with low air resistance
- Every little bit, birds need a rest
- If birds see food, they will probably go eat it

Flocking Simulation

What do birds do?

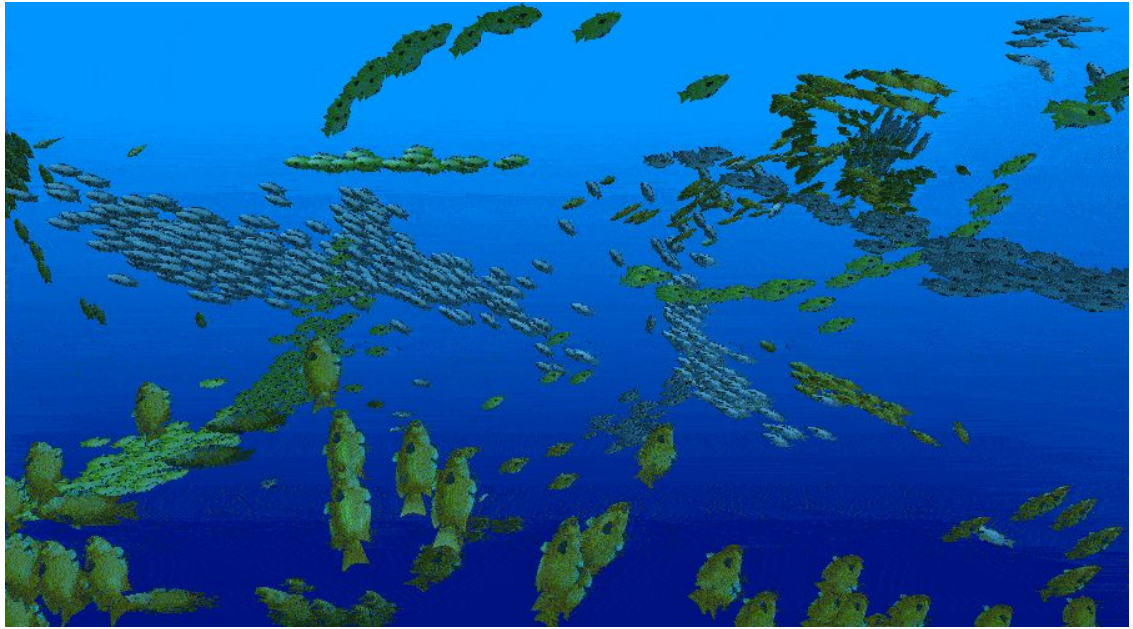
- **Birds like to follow each other**
- **Birds don’t want to be too close to each other**
- **When flying, birds look for spaces with low air resistance**
- Every little bit, birds need a rest
- If birds see food, they will probably go eat it



Flocking Simulation

What do fish do?

- Fish like to follow each other
- Fish don’t want to be too close to each other
- Fish like to swim with fish of the same species



Flocking Simulation

What do zombies do?

Flocking Simulation

What do zombies do?

- Zombies can only walk on a surface, or climb on a surface with over 30% elevation
- Zombies aren't very good at climbing
- When zombies fall, they take a minute, then get back up and keep going
- Zombies want to get to the other side of the wall

Flocking Simulation

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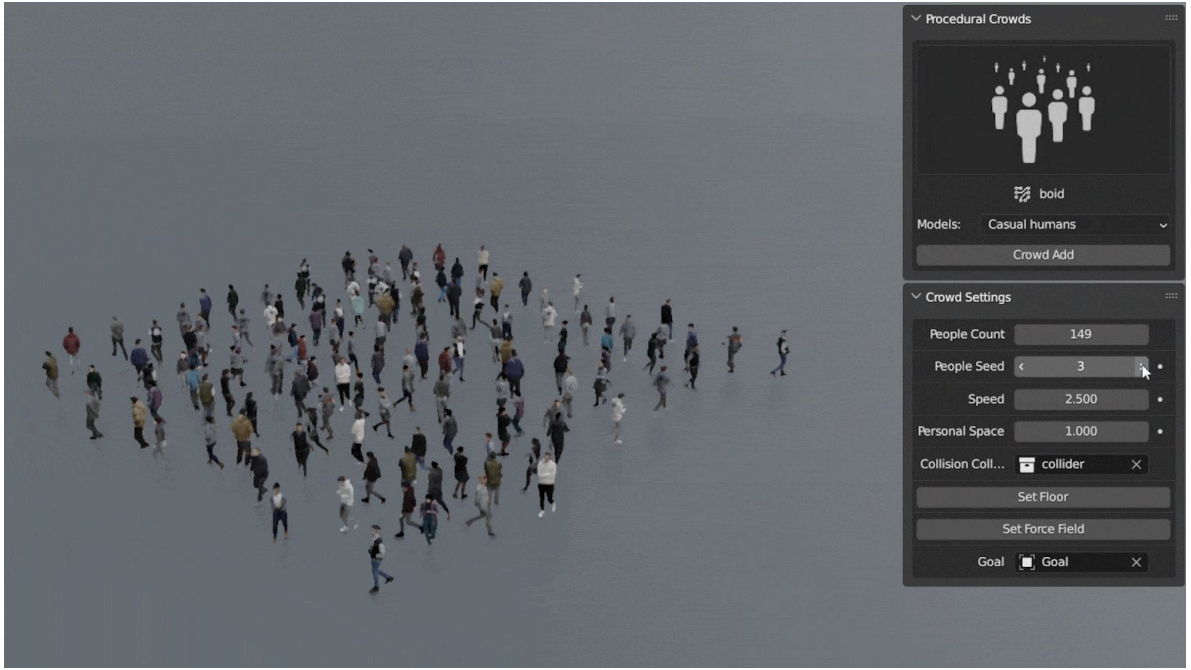
- Zombies can only walk on a surface, or climb on a surface with over 30% elevation
- Zombies aren't very good at climbing
- When zombies fall, they take a minute, then get back up and keep going
- Zombies want to get to the other side of the wall

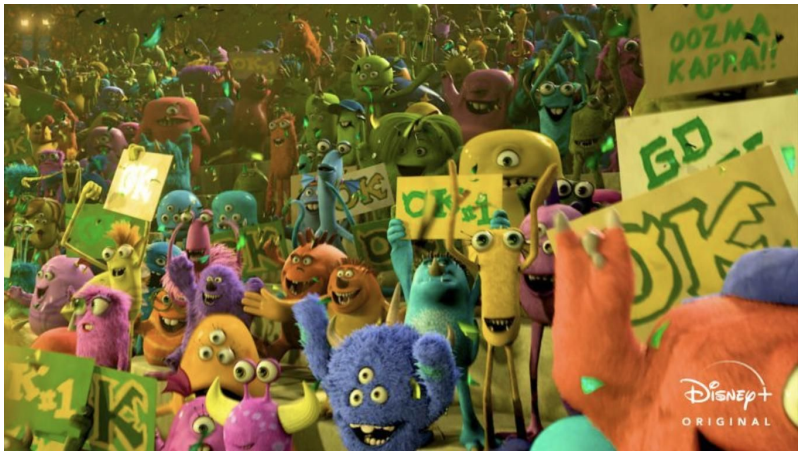
Flocking Simulation

What do people do?

- People can only walk on surfaces
- Often people have a destination in mind
- People don’t want to be too close to strangers



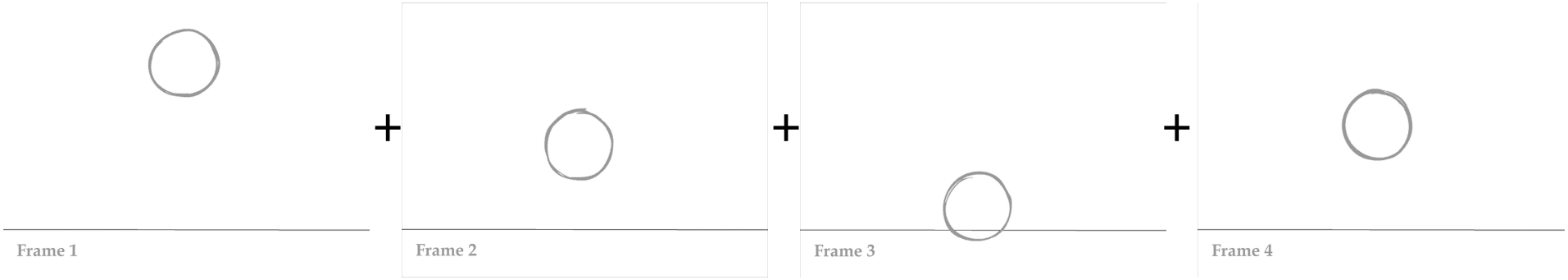
Flocking Simulation



Additional Resources + Sources

- Collisions – Blender 5.2
- Fluid Simulation Research Papers
- Sag-Fee Hair Simulation (University of Utah!)
- Python Numerical Methods (Implicit/Explicit Combinations)
- Check out extra slides at the end for simulation math examples!

Simulation



Envato Tuts Animation tutorial

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Simulation

F = -9.81
V = 0
P = .04

A = -9.81
V_next = -.39
P_next = .024

Frame 1

+

F = -9.81
V = -.39
P = .024

A = -9.81
V_next = -.78
P_next = -.007

Frame 2

+

F = -9.81 **+ momentum**
V = -.78
P = 0

A = **N/A**
V_next = **+**.78
P_next = .031

Frame 3

+

F = -9.81
V = .78
P = .031

A = -9.81
V_next = .39
P_next = .04

Frame 4

Envato Tuts Animation tutorial

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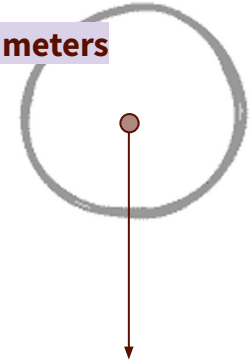
Frame 1

Simulation Math

old velocity = (0, 0) meters/second

old position = (0, .04) meters

Mass = 1.0 g



Force = (0, -9.81 * mass)N

time step = .04 seconds

acceleration = force / mass

change in Velocity = acceleration * time step

New velocity = old velocity + change in velocity

change in position = new velocity * time step

New position = Old Position + change in position

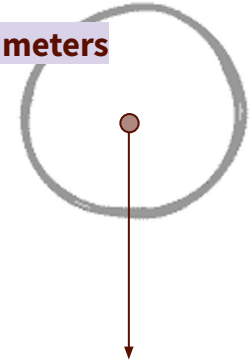
Frame 1

Simulation Math

old velocity = (0, 0) meters/second

old position = (0, .04) meters

Mass = 1.0 g



Force = (0, -9.81 * mass)N

time step = .04 seconds

acceleration = force / mass
acceleration = (0, -9.81 * 1.0g) N / 1.0g
acceleration = (0, -9.81) m/s^2

change in Velocity = acceleration * time step

New velocity = old velocity + change in velocity

change in position = new velocity * time step

New position = Old Position + change in position

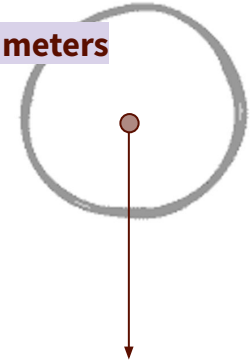
Frame 1

Simulation Math

old velocity = (0, 0) meters/second

old position = (0, .04) meters

Mass = 1.0 g



Force = (0, -9.81 * mass)N

time step = .04 seconds

acceleration = force / mass
acceleration = (0, -9.81 * 1.0g) N / 1.0g
acceleration = (0, -9.81) m/s^2

change in Velocity = acceleration * time step
change in Velocity = (0, -9.81)m/s^2 * .04s
change in Velocity = (0, -.39)m/s

New velocity = old velocity + change in velocity

change in position = new velocity * time step

New position = Old Position + change in position

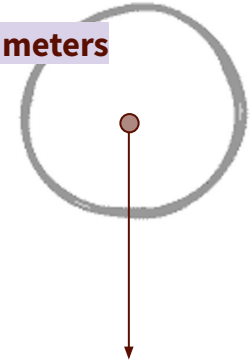
Frame 1

Simulation Math

old velocity = (0, 0) meters/second

old position = (0, .04) meters

Mass = 1.0 g



Force = (0, -9.81 * mass)N

time step = .04 seconds

acceleration = force / mass
acceleration = (0, -9.81 * 1.0g) N / 1.0g
acceleration = (0, -9.81) m/s^2

change in Velocity = acceleration * time step
change in Velocity = (0, -9.81)m/s^2 * .04s
change in Velocity = (0, -.39)m/s

New velocity = old velocity + change in velocity
New velocity = (0,0)m/s + (0, -.39)m/s
New velocity = (0, -.39)m/s

change in position = new velocity * time step

New position = Old Position + change in position

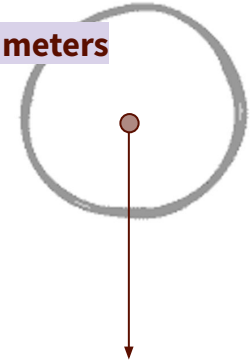
Frame 1

Simulation Math

old velocity = (0, 0) meters/second

old position = (0, .04) meters

Mass = 1.0 g



Force = (0, -9.81 * mass)N

time step = .04 seconds

acceleration = force / mass
acceleration = (0, -9.81 * 1.0g) N / 1.0g
acceleration = (0, -9.81) m/s^2

change in Velocity = acceleration * time step
change in Velocity = (0, -9.81)m/s^2 * .04s
change in Velocity = (0, -.39)m/s

New velocity = old velocity + change in velocity
New velocity = (0,0)m/s + (0, -.39)m/s
New velocity = (0, -.39)m/s

change in position = new velocity * time step
change in position = (0, -.39)m/s * .04s
change in position = (0, -.016)m

New position = Old Position + change in position

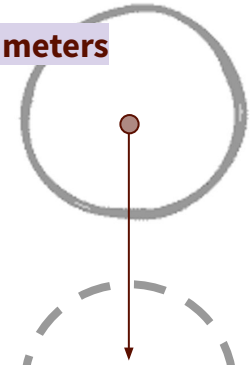
Frame 1

Simulation Math

old velocity = (0, 0) meters/second

old position = (0, .04) meters

Mass = 1.0 g



Force = (0, -9.81 * mass)N

time step = .04 seconds

acceleration = force / mass
acceleration = (0, -9.81 * 1.0g) N / 1.0g
acceleration = (0, -9.81) m/s^2

change in Velocity = acceleration * time step
change in Velocity = (0, -9.81)m/s^2 * .04s
change in Velocity = (0, -.39)m/s

New velocity = old velocity + change in velocity
New velocity = (0,0)m/s + (0, -.39)m/s
New velocity = (0, -.39)m/s

change in position = new velocity * time step
change in position = (0, -.39)m/s * .04s
change in position = (0, -.016)m

New position = Old Position + change in position
New position = (0, .04) m + (0, -.016)m
New position = (0, .024)m

Frame 1

Simulation Math

F = -9.81
V = 0
P = .04

A = -9.81
V_next = -.39
P_next = .024

Frame 1

+

F = -9.81
V = -.39
P = .024

A = -9.81
V_next = -.78
P_next = -.007

Frame 2

+

F = -9.81 + momentum
V = -.78
P = 0

A = N/A
V_next = +.78
P_next = .031

Frame 3

+

F = -9.81
V = .78
P = .031

A = -9.81
V_next = .39
P_next = .04

Frame 4

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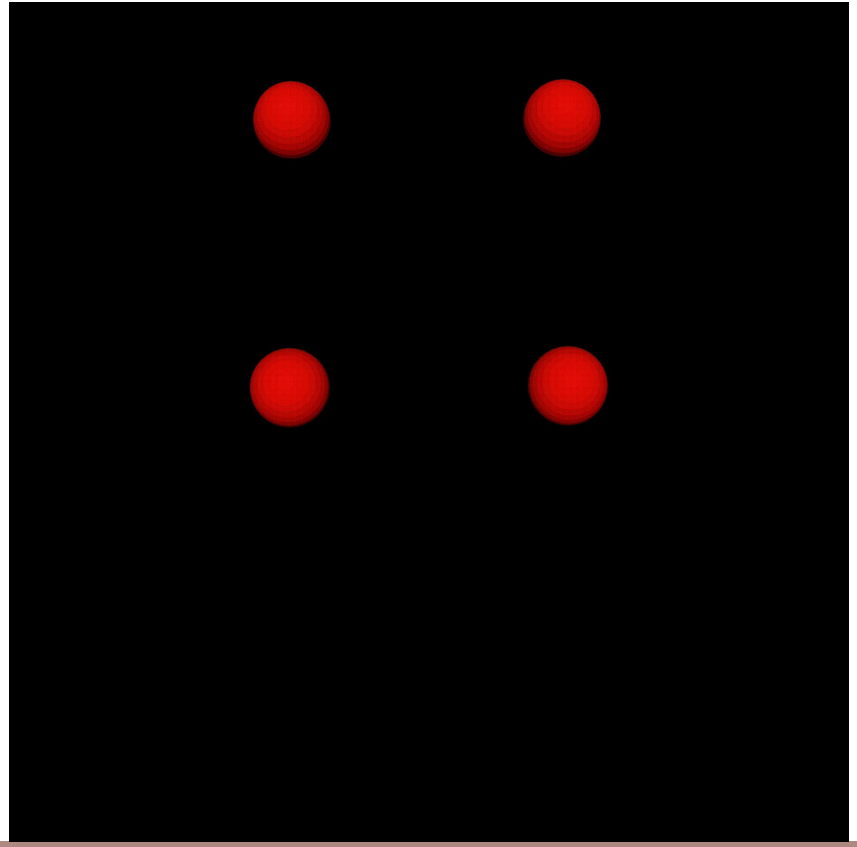
Frame 1

Gravity Simulation

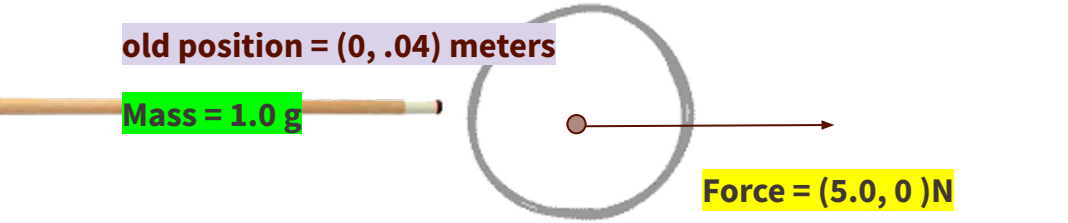


Simulation Math Part 2

old velocity = (0, 0) meters/second

old position = (0, .04) meters

Mass = 1.0 g



Force = (5.0, 0)N

time step = .04 seconds

acceleration = force / mass

change in Velocity = acceleration * time step

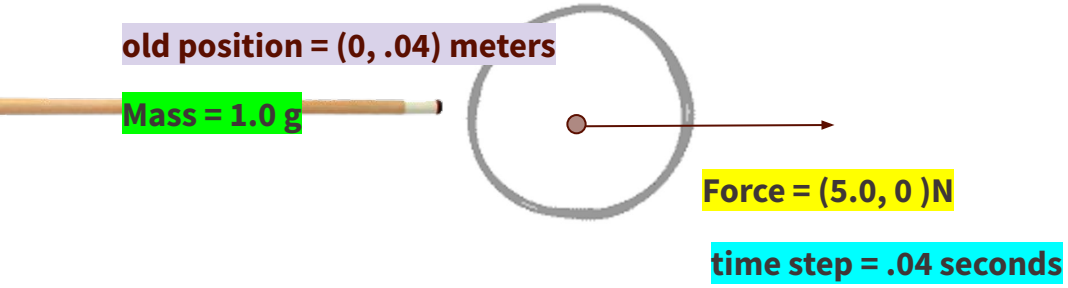
New velocity = old velocity + change in velocity

change in position = new velocity * time step

New position = Old Position + change in position

Frame 1

Simulation Math Part 2



acceleration = force / mass
acceleration = (5.0, 0) N / 1.0g
acceleration = (5.0, 0) m/s^2

change in Velocity = acceleration * time step

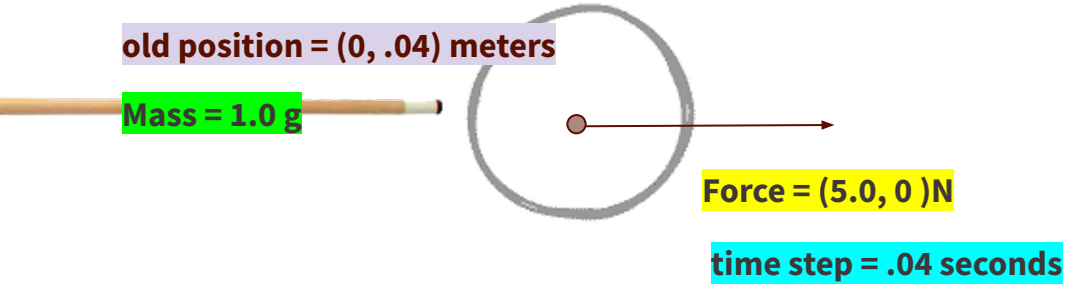
New velocity = old velocity + change in velocity

change in position = new velocity * time step

New position = Old Position + change in position

Frame 1

Simulation Math Part 2



acceleration = force / mass
acceleration = (5.0, 0) N / 1.0g
acceleration = (5.0, 0) m/s^2

change in Velocity = acceleration * time step
change in Velocity = (5.0, 0)m/s^2 * .04s
change in Velocity = (.2, 0)m/s

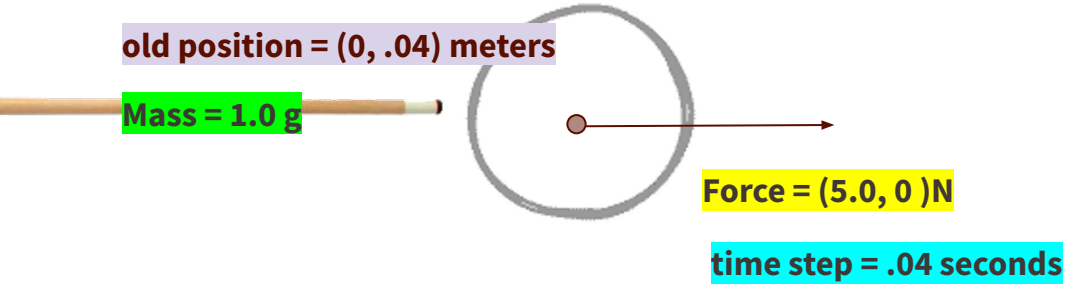
New velocity = old velocity + change in velocity

change in position = new velocity * time step

New position = Old Position + change in position

Frame 1

Simulation Math Part 2



acceleration = force / mass
acceleration = (5.0, 0) N / 1.0g
acceleration = (5.0, 0) m/s^2

change in Velocity = acceleration * time step
change in Velocity = (5.0, 0)m/s^2 * .04s
change in Velocity = (.2, 0)m/s

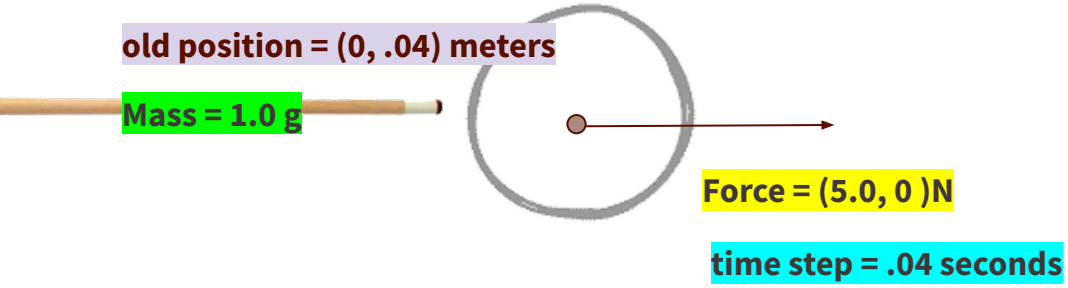
New velocity = old velocity + change in velocity
New velocity = (0,0)m/s + (.2, 0)m/s
New velocity = (.2, 0)m/s

change in position = new velocity * time step

New position = Old Position + change in position

Frame 1

Simulation Math Part 2



acceleration = force / mass
acceleration = (5.0, 0) N / 1.0g
acceleration = (5.0, 0) m/s^2

change in Velocity = acceleration * time step
change in Velocity = (5.0, 0)m/s^2 * .04s
change in Velocity = (.2, 0)m/s

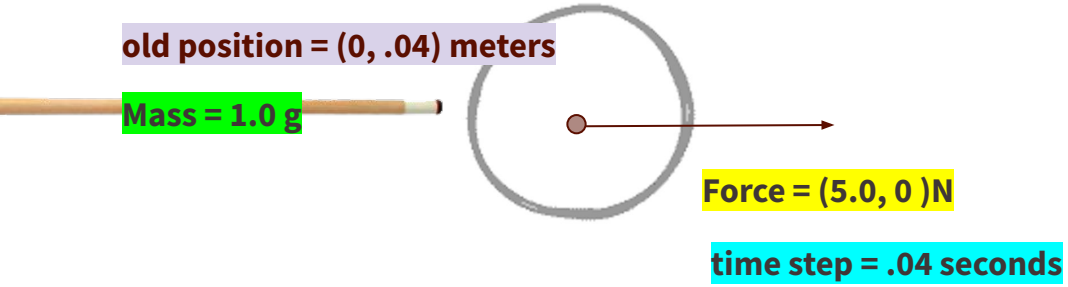
New velocity = old velocity + change in velocity
New velocity = (0,0)m/s + (.2, 0)m/s
New velocity = (.2, 0)m/s

change in position = new velocity * time step
change in position = (.2, 0)m/s * .04s
change in position = (.008, 0)m

New position = Old Position + change in position

Frame 1

Simulation Math Part 2



acceleration = force / mass
acceleration = (5.0, 0) N / 1.0g
acceleration = (5.0, 0) m/s^2

change in Velocity = acceleration * time step
change in Velocity = (5.0, 0)m/s^2 * .04s
change in Velocity = (.2, 0)m/s

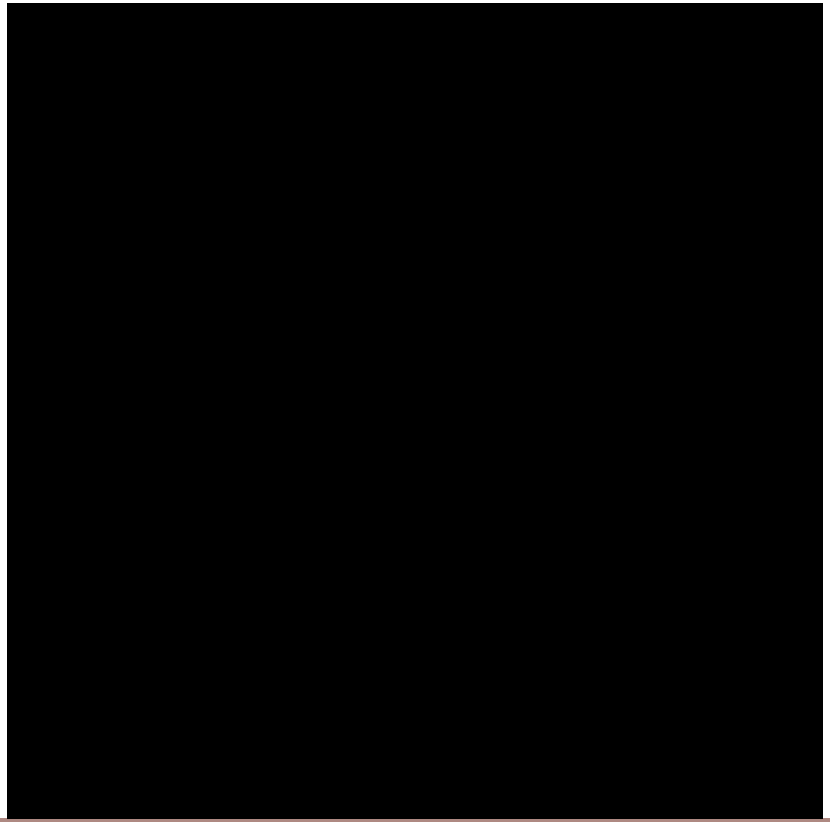
New velocity = old velocity + change in velocity
New velocity = (0,0)m/s + (.2, 0)m/s
New velocity = (.2, 0)m/s

change in position = new velocity * time step
change in position = (.2, 0)m/s * .04s
change in position = (.008, 0)m

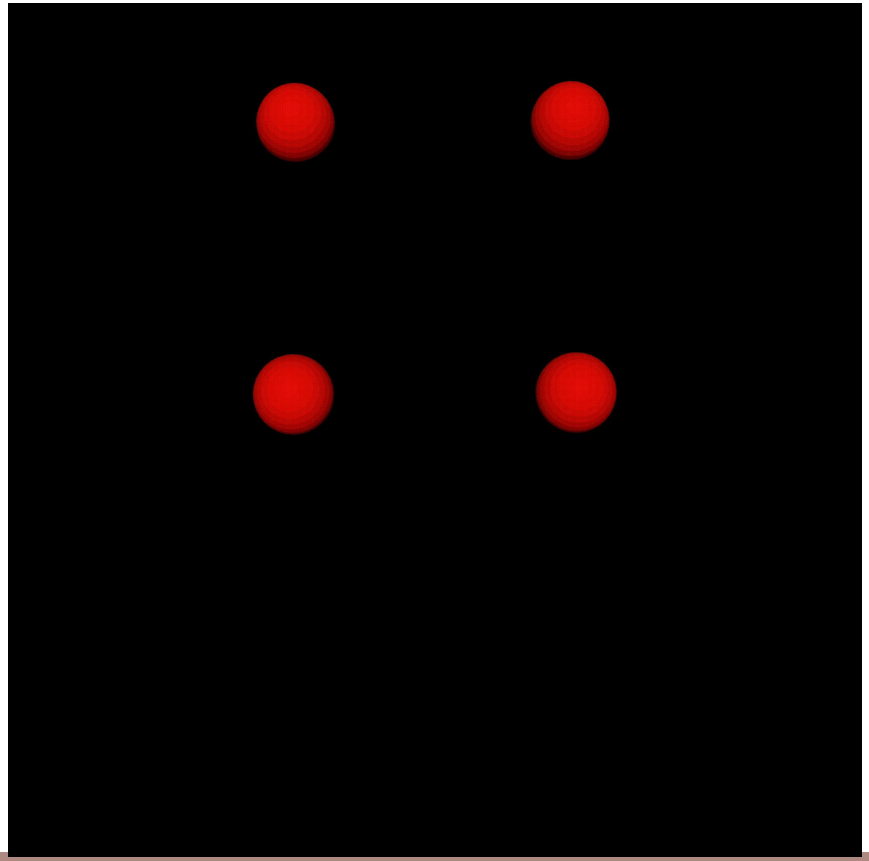
New position = Old Position + change in position
New position = (0, .04) m + (.008, 0)m
New position = (.008, .04)m

Frame 1

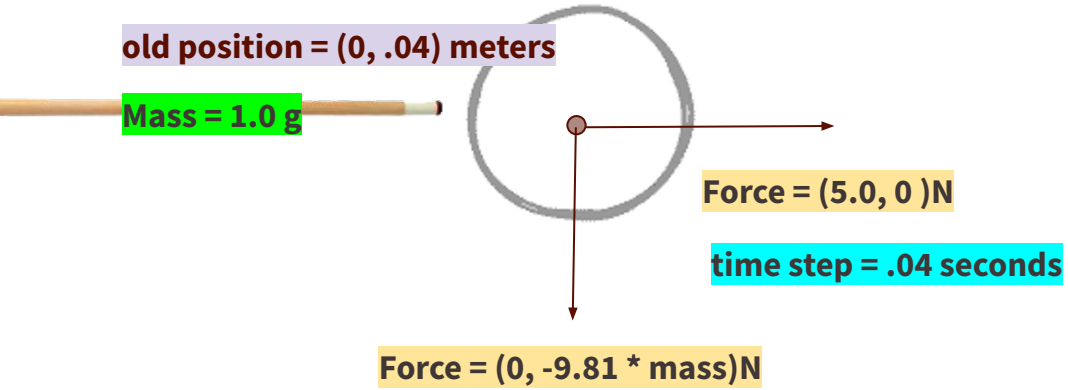
Attraction Simulation



Attraction Simulation



Simulation Math Multiple Forces



acceleration = force / mass

change in Velocity = acceleration * time step

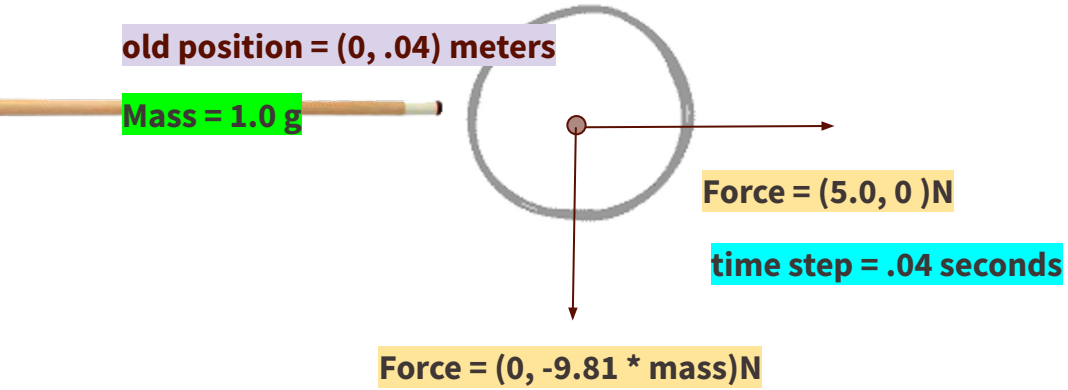
New velocity = old velocity + change in velocity

change in position = new velocity * time step

New position = Old Position + change in position

Frame 1

Simulation Math Multiple Forces



acceleration = force / mass
acceleration = (5.0, -9.81* 1.0g) N / 1.0g
acceleration = (5.0, -9.81) m/s^2

change in Velocity = acceleration * time step

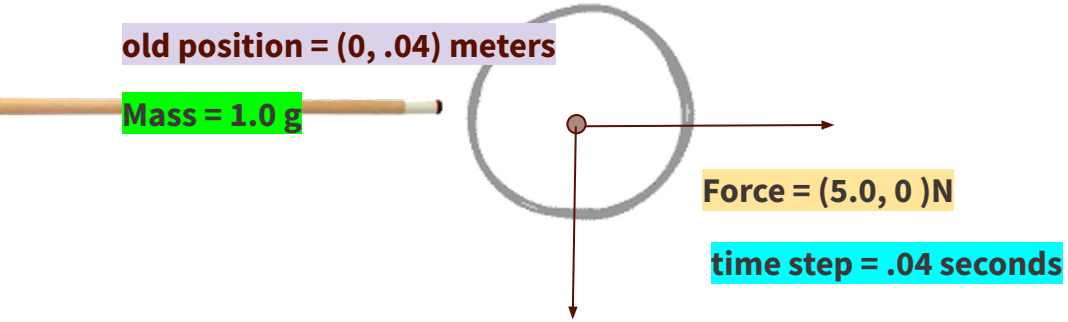
New velocity = old velocity + change in velocity

change in position = new velocity * time step

New position = Old Position + change in position

Frame 1

Simulation Math Multiple Forces



acceleration = force / mass
acceleration = (5.0, -9.81* 1.0g) N / 1.0g
acceleration = (5.0, -9.81) m/s^2

change in Velocity = acceleration * time step
change in Velocity = (5.0, -9.81)m/s^2 * .04s
change in Velocity = (.2, -.39)m/s

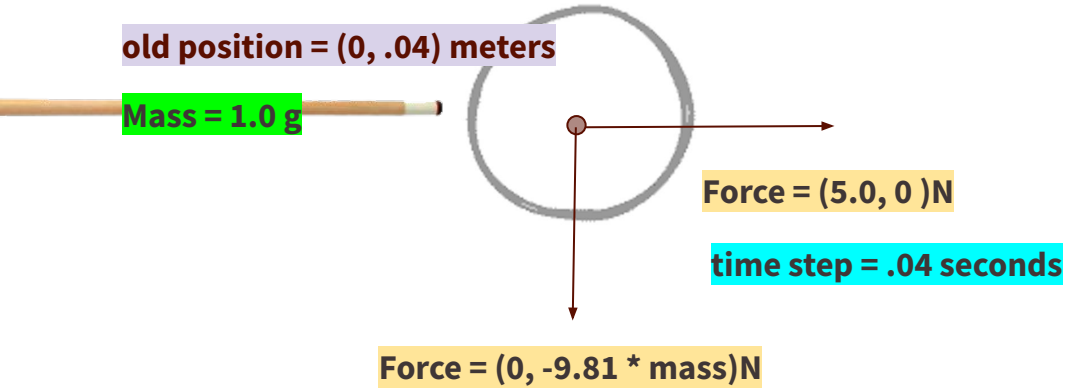
New velocity = old velocity + change in velocity

change in position = new velocity * time step

New position = Old Position + change in position

Frame 1

Simulation Math Multiple Forces



acceleration = force / mass
acceleration = (5.0, -9.81* 1.0g) N / 1.0g
acceleration = (5.0, -9.81) m/s^2

change in Velocity = acceleration * time step
change in Velocity = (5.0, -9.81)m/s^2 * .04s
change in Velocity = (.2, -.39)m/s

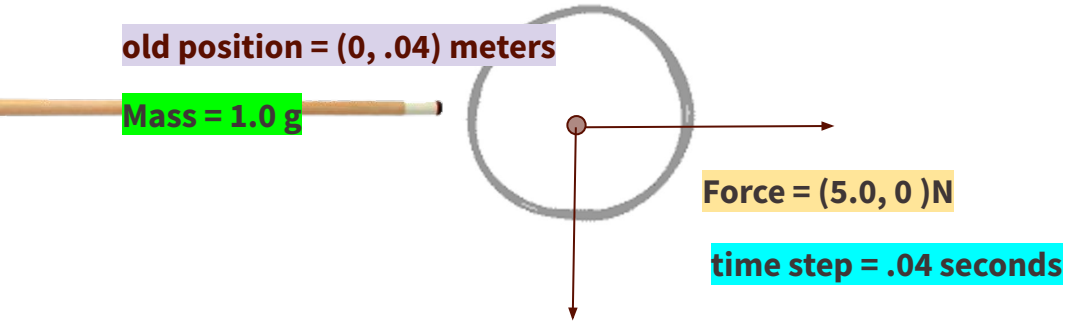
New velocity = old velocity + change in velocity
New velocity = (0,0)m/s + (.2, -.39)m/s
New velocity = (.2, -.39)m/s

change in position = new velocity * time step

New position = Old Position + change in position

Frame 1

Simulation Math Multiple Forces



acceleration = force / mass
acceleration = (5.0, -9.81* 1.0g) N / 1.0g
acceleration = (5.0, -9.81) m/s^2

change in Velocity = acceleration * time step
change in Velocity = (5.0, -9.81)m/s^2 * .04s
change in Velocity = (.2, -.39)m/s

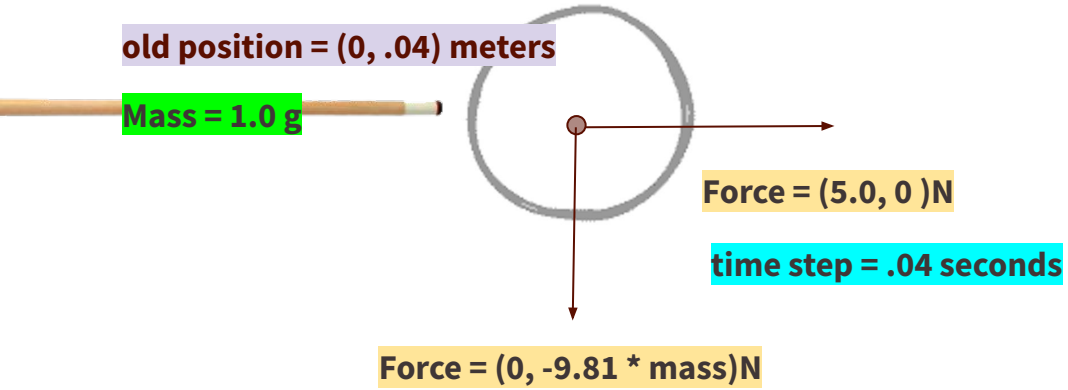
New velocity = old velocity + change in velocity
New velocity = (0,0)m/s + (.2, -.39)m/s
New velocity = (.2, -.39)m/s

change in position = new velocity * time step
change in position = (.2, 0)m/s * .04s
change in position = (.008, -.016)m

New position = Old Position + change in position

Frame 1

Simulation Math Multiple Forces



acceleration = force / mass
acceleration = (5.0, -9.81* 1.0g) N / 1.0g
acceleration = (5.0, -9.81) m/s^2

change in Velocity = acceleration * time step
change in Velocity = (5.0, -9.81)m/s^2 * .04s
change in Velocity = (.2, -.39)m/s

New velocity = old velocity + change in velocity
New velocity = (0,0)m/s + (.2, -.39)m/s
New velocity = (.2, -.39)m/s

change in position = new velocity * time step
change in position = (.2, 0)m/s * .04s
change in position = (.008, -.016)m

New position = Old Position + change in position
New position = (0, .04) m + (.008, -.016)m
New position = (.008, .024)mo

Frame 1

Multiple Forces Simulation